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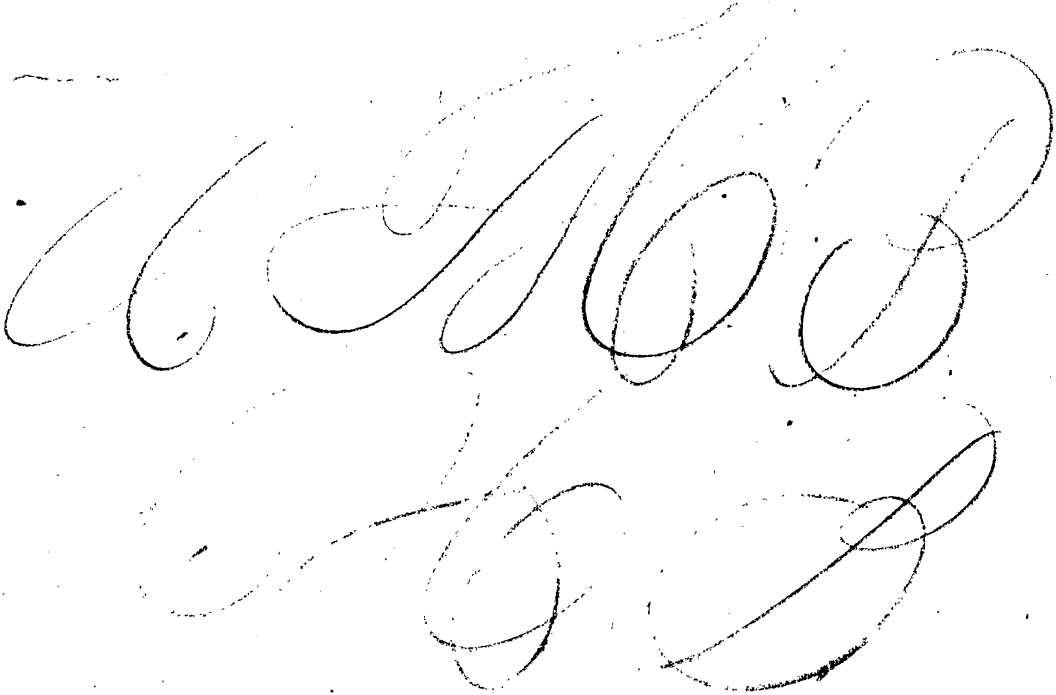

ELEMENTARY ARITHMETIC;

OR,

SECOND BOOK

UNIV. OF CALIFORNIA

OF A

SERIES OF MATHEMATICS.

BY

ANDREW H. BAKER, A.M., PH.D.,

REVISED AND ENLARGED EDITION.

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PREFACE.

THIS book contains the Elements of Arithmetic, or perhaps more properly the Elements of Mathematical Science.

In its preparation I have endeavored to give it not only a mathematical basis, but also a scientific structure. This I have done, keeping steadily in mind that it is a book for the young, for whom the pathway of science should be made as easy and inviting as possible.

We cannot insist too strongly on the advantages of the blackboard exercises in developing the principles of science, and rendering them more easily and more thoroughly understood. In a well-drilled class, whilst one student is at the board demonstrating a theorem or solving a problem, and the other members of the class are looking on with attention, the latter learn as much as the former.

The teacher should be the guide of the class. He should not be satisfied with exemplification only, he should also endeavor to encourage and interest his pupils, carefully observing the progress made each day, each week, each month, and at the end of the year when all shall be summed up, such will be the delight of both teacher and class at the progress made, that they will begin to believe there is a royal road to learning.

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The Author cannot too highly recommend to the Teacher the use of the Blackboard described on the following page. Great facility in comprehending the combinations and divisions of numbers will be acquired by this method.

BLACKBOARD EXERCISE.

1 This page represents a blackboard with the num- 37
2 bers as high as 72 painted on its margins. 38
3 There is also a box containing slips which will cover 39
4 two, three, four, etc., as high as 12, and numbered 40
5 accordingly; one of these the student will take in his 41
6 hand and apply it to the painted numbers to perform 42
7 addition or subtraction; thus, begin at 1 and take a 43
8 slip marked 2, then 1 and 2 are 3, 3 and 2 are 5, 5 and 44
9 2 are 7, 7 and 2 are 9, etc., counting at least the left- 45
10 hand column; then, to perform subtraction, begin at 46
11 the bottom of the 1st column; thus, 36 minus 2 equals 47
12 34, 34−2=32, 32−2=30, 30−2=28, etc., until the top 48
13 is reached; then taking a slip marked 3, begin with 49
14 1 or 2, or first with 1 and then with 2, and return to 50
15 the top of the column as before, by subtraction; let 51
16 this exercise be performed with all the slips, and as 52
17 the larger numbers are taken, continue the additions 53
18 to the bottom of the 2d column, and return as before. 54
19 For multiplication and division first make a chalk 55
20 mark after every two figures up to 24, and mul- 56
21 tiply; thus, once 2 are 2, twice 2 are 4, 3 times 2 are 57
22 6, 4 times 2 are 8, etc.; then the number of divisions 58
23 is 12 and each division has 2 numbers; ∴ 12 is con- 59
24 tained twice in 24, or 2 is contained 12 times, 2 is 60
25 contained once in 2, in 4 twice, in 6 three times, in 8 61
26 four times, in 10 five times, in 12 six times, etc. When 62
27 the student is familiar with multiplication and division 63
28 by 2, let the numbers be separated into 3's, then 4's, 64
29 etc., and let each be continued for 12 divisions; when 65
30 all the divisions have been performed according to the 66
31 steps, beginning with 2 and ending with 12, a multi- 67
32 plication and division table will be made. 68
33 REM.—In multiplication the product of any two fac- 69
34 tors is the same by making either the multiplicand and 70
35 the other the multiplier; so also in division, the divisor
and the quotient may be substituted, as the dividend 71
36 is the product of the divisor and quotient. 72

REM.—The numbers, continued up to 144, should be painted on the sides of the board.

ELEMENTARY ARITHMETIC.

DEFINITIONS.

1. ***Arithmetic*** is the science of numbers.

2. A ***Unit*** is a single thing; as, a book, one dollar, or simply one.

3. A ***Number*** is a unit or a collection of units; as, one, ten, five books, twenty-five dollars.

4. The numbers used in Arithmetic are all formed by combinations of the ten Arabic characters, called ***Figures;*** viz., 0, called zero or naught; 1, called one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine.

5. Expressing a number either in writing or figures is called ***Notation,*** and reading the expression is called ***Numeration.***

6. When numbers are used without reference to any object, they are called ***Abstract Numbers;*** as, five, twenty, etc.; but when they are applied to things, they are called ***Concrete;*** as, one book, ten men, four dollars, etc.

7. When concrete numbers express values of money, weights, measures, time, etc., they are called ***Denomi-***

nate Numbers; as, dollars, pounds, shillings, pounds of weight, ounces, hours, minutes, etc.

8. When different denominations of either kind form but one number, it is called a ***Compound Number;*** as, £4 3s. 6d., 2 lb. 1 oz. 3 pwt. and 2 gr.

9. Numbers of the same order and the same denomination are termed ***Like Numbers;*** other numbers are termed ***Unlike Numbers.***

REM.—Numbers expressing different species of the same genus are unlike, as horses and cows; while the same numbers expressed in the term of the genus are alike, as animals.

MATHEMATICAL TERMS USED IN ARITHMETIC.

1. An affirmative sentence, or anything proposed for consideration, is a ***Proposition.***

2. A self-evident proposition is called an ***Axiom.***

3. A proposition made evident by a demonstration is called a ***Theorem.***

4. When a proposition is used for developing a principle of Arithmetic, it is called a ***Problem.***

5. Propositions given merely for solution, in order to impress the principles on the mind, are called ***Examples.***

6. An obvious consequence of one or more propositions is called a ***Corollary.***

7. An established custom, or an assumption without proof, is called a ***Postulate.***

REM. 1 and 1 are 2, 2 and 1 are 3, 3 and 1 are 4, 5 and 2 are 7, 6 and 3 are 9, etc., is the postulate which forms the basis of Arithmetic.

AXIOMS.

1. If equal numbers are added to equal numbers, the sums will be equal.

2. If equal numbers are subtracted from equal numbers, the remainders will be equal.

3. If equals be multiplied by equals, the products will be equal.

4. If equals be divided by equals, the quotients will be equal.

5. If two numbers are each equal to the same number, they are equal to each other.

6. If the same number be added to and subtracted from another number, the latter number will not be changed.

7. If a number be both multiplied and divided by the same number, the former number will not be changed.

8. If two numbers be equally increased or diminished, the difference of the resulting numbers will be the same as the difference of the originals.

9. If two numbers are like parts of equal numbers, they are equal to each other.

10. The whole is greater than any of its parts.

11. The whole is equal to the sum of all its parts.

SIGNS.

1. The sign +, called ***plus***, is the sign of addition, and indicates that the number on the right hand is to be added to the one on the left.

2. The sign —, called ***minus***, is the sign of subtraction, and indicates that the number on the right is to be subtracted from that on the left.

3. The sign ×, called ***into***, is the sign of multiplication, and indicates that the numbers between which it is placed are factors of the same product.

4. The sign ÷, ***divided by***, the left-hand number to be divided by the right hand.

5. The sign =, ***equal to***, indicates that the numbers between which it is placed are equal.

6. 5^2, 5^3, the 2 and 3 placed to the right, a little above a number, indicates the ***power*** to which it is to be raised.

7. $\sqrt{\ }$, indicates the extraction of the ***square root;*** and $\sqrt[3]{\ }$, indicates the extraction of the ***cube root.***

NOTATION AND NUMERATION.

1st. A figure standing alone, as 1, 2, 3, holds the units place, or is of the 1st order, and is read, *one, two, three.*

2d. A number having two figures, as 14, 26, the right-hand figure holds the units place, and the left-hand figure that of tens, and they are read, *fourteen, twenty-six.*

COR.—The right-hand figure of a number is called *units*, or the 1st order; the next figure to the left is called *tens*, or the 2d order; the third figure, *hundreds*, or the 3d order; the fourth figure, *thousands*, or the 4th order; and if a number be expressed with the nine figures in order, making 1 the right-hand figure, the figures will express their respective orders; thus,

millions,			thousands,			units.		
hundreds of	tens of	units of	hundreds of	tens of	units of	hundreds of	tens of	units of
9	8	7,	6	5	4,	3	2	1

If pointed in periods of three figures each, they may be read as follows: *Nine hundred and eighty-seven millions six hundred and fifty-four thousand three hundred and twenty-one.*

Rem.—The figures designate the orders.

Read the following numbers:

1	one.
21	twenty-one.
321	three hundred and twenty-one.
4,321	four thousand three hundred and twenty-one.
54,321	fifty-four thousand three hundred and twenty-one.
654,321	six hundred and fifty-four thousand three hundred and twenty-one.
7,654,321	seven millions six hundred and fifty-four thousand three hundred and twenty-one.
87,654,321	eighty-seven millions six hundred and fifty-four thousand three hundred and twenty-one.
987,654,321	nine hundred and eighty-seven millions six hundred and fifty-four thousand three hundred and twenty-one.

Rem.—The column of 1's is of the 1st order, the column of 2's is of the 2d order, the 3's the 3d order, the 4's the 4th order, etc.

Cor.—The relation of any two consecutive orders is the same, for when in addition the sum of any column reaches 10, the left-hand figure belongs to the next column or order; hence, a table may be formed, thus,

10 units = 1 ten.
10 tens = 1 hundred.
10 hundred = 1 thousand.
10 thousand = 1 ten-thousand.
10 ten-thousand = 1 hundred-thousand.
10 hundred-thousand = 1 million.
etc. etc.

ADDITION AND SUBTRACTION.

ADDITION AND SUBTRACTION TABLE.

0	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18
10	11	12	13	14	15	16	17	18	19
11	12	13	14	15	16	17	18	19	20
12	13	14	15	16	17	18	19	20	21
13	14	15	16	17	18	19	20	21	22
14	15	16	17	18	19	20	21	22	23
15	16	17	18	19	20	21	22	23	24
16	17	18	19	20	21	22	23	24	25
17	18	19	20	21	22	23	24	25	26
18	19	20	21	22	23	24	25	26	27
19	20	21	22	23	24	25	26	27	28
20	21	22	23	24	25	26	27	28	29
21	22	23	24	25	26	27	28	29	30

The square made by the ten Arabic characters forms an Addition and Subtraction Table.

Beginning with the first line, thus, Zero and zero are zero; zero and 1 are 1; zero and 2 are 2; zero and 3 are 3; zero and 4 are 4; zero and 5 are 5; zero and 6 are 6, etc.

The second line, one and zero are one; 1 and 1 are 2: 1 and 2 are 3; 1 and 3 are 4; 1 and 4 are 5, etc.

2 and zero are 2; 2 and 1 are 3; 2 and 2 are 4; 2 and 3 are 5; 3 and 4 are 7, etc.

3 and 0 are 3; 3 and 1 are 4; 3 and 2 are 5; 3 and 3 are 6; 3 and 4 are 7, etc.

Continue this, taking the first figure of the 1st column and adding it to each successive figure in the first line; the adding of zero is only nominal, as it makes no increase.

It also becomes a subtraction table, the figures of the first column being the subtrahend, and those of the first line the remainders.

Take zero from 1 and 1 remains; 0 from 2, 2 remain, etc. It may be thus expressed: $1 - 0 = 1$; $2 - 0 = 2$; $3 - 0 = 3$; $4 - 0 = 4$; $5 - 0 = 5$; which is read, 1 minus zero equals 1, etc.

Second line: take 1 from 2, 1 remains; 1 from 3, 2 remain; or, $2 - 1 = 1$; $3 - 1 = 2$; $4 - 1 = 3$; $5 - 1 = 4$, etc.

Third line; $3 - 2 = 1$; $4 - 2 = 2$; $5 - 2 = 3$; $6 - 2 = 4$; $7 - 2 = 5$, etc.

In addition, we add two numbers at a time, never more, and in the first square we have the addition of every two units that can come together; so also in subtraction.

In the second square, the units correspond with the first square, and have an additional ten.

In the third square, the units again are repeated, and another additional ten.

As a column of tens, hundreds, and every higher or lower order is added and subtracted in the same way, the above table develops every principle of addition and subtraction.

Add the column of units.

$$\begin{array}{rr} 1 & 3 \\ 2 & 5 \\ 3 & 6 \\ 4 & 4 \\ 5 & 2 \\ 6 & 7 \\ 7 & 9 \\ \hline 28 & 36 \end{array}$$

One and 2 are 3; 3 and 3 are 6; 6 and 4 are 10; 10 and 5 are 15; 15 and 6 are 21; 21 and 7 are 28; or as is customary to begin at the bottom of the column, 7 and 6 are 13; 13 and 5 are 18; 18 and 4 are 22; 22 and 3 are 25; 25 and 2 are 27; 27 and 1 are 28.

$9 + 7 = 16$; $16 + 2 = 18$; $18 + 4 = 22$; $22 + 6 = 28$; $28 + 5 = 33$; $33 + 3 = 36$.

Rem.—Although many numbers may be added together, in performing the operation only two at a time are added.

Add the following numbers jointly and separately; thus,

$$\begin{array}{rcrcrcr} 35 & = & 30 & \text{and} & 5 & & \\ 24 & = & 20 & \text{“} & 4 & & \\ 43 & = & 40 & \text{“} & 3 & & \\ 52 & = & 50 & \text{“} & 2 & & 21 \\ 67 & = & 60 & \text{“} & 7 & & 200 \\ \hline 221 & & 200 & \text{“} & 21 & = & 221 \end{array}$$

The sum of the column of units is 21; that is, 1 unit and 2 tens; the sum of the column of tens is 20; that is, 20 tens or 2 hundred; and the two sums united make

221; precisely the same as if the column of units is first added, and the units of the sum placed under the column of units, and the tens added with the column of tens; and then the tens of the sum of the tens column placed under the column of tens, and the hundreds in place of hundreds.

Cor.—As the relation of each successive order is the same, hence for every ten of any order, the 1, or left-hand figure, belongs to the next order; and the process is the same in the addition of every column; that is, one is carried to the next column for every ten in the addition of each column.

ADDITION.			SUBTRACTION.	
3241	4365	643215	876432	987654
4356	5321	532684	543210	321234
6745	7546	478921		
5364	8432	586432	876543	789654
19706			654321	321043
754321	864321	345678	869754	
678643	678963	987654	654321	
594721	987654	321987		
367543	456789	654321		

Add

7654	3465321
3897	6354789
11551	9820110

Subtract

Minuend,	11551	*Minuend,*	11551
Subtrahend,	7654	*Subtrahend,*	3897
	3897		7654

Minuend,	9820110	*Minuend,*	9820110
Subtrahend,	3465321	*Subtrahend,*	6354789
	6354789		3465321

Cor. 1.—The minuend is always equal to the sum of the subtrahend and remainder, and is therefore greater than either.

Cor. 2.—Arithmetic is based upon the postulate contained in 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, *which is addition,* etc.; and the application of Axiom 6 (page 9) to this postulate proves the principle of subtraction; thus, $19 + 6 = 25$, then $25 - 6$ must be equal to 19.

Rem. 1.—When the figure of the subtrahend is larger than the one above it of the same order of the minuend, 1 of the next order of the minuend must be united to the figure of the minuend and then the subtraction be performed; then in order to make up for this addition to the minuend, 1 must be added to the next order of the subtrahend, and then perform the subtraction; this process is called carrying and requires all the attention of the student.

Rem. 2.—I prefer few examples, but these may be often repeated, and if thought necessary the teacher can give others in which the columns are longer.

Rem. 3.—Each order may be regarded as units, and the sum may reach one, two, or more hundred of its order.

PRACTICAL EXAMPLES

1. A farmer has 17 sheep in one pasture, 41 in another, and 57 in a third; how many sheep has he?

2. A fowler has 18 turkeys, 21 geese, 42 ducks, and 64 chickens; how many fowls has he?

3. A man has 48 cattle in one field and 36 in another;

if he take 15 from the second field and put them in the first, how many will then be in each field?

4. A granger owns 120 cattle, which are pastured in three fields; in one field there are 45 cattle, in another 30 cattle; how many cattle in the third field?

5. Willie has 41 cents in one pocket and 37 in another; if he buy a knife for 50 cents, how many cents will he have left?

6. Willie had given him two books to read, one of 153 pages and the other of 226 pages; he has read 240 pages, how many more has he to read?

7. In one basket there are 51 eggs, in another 62, and in a third 42 eggs; how many in the three baskets?

8. One hen has 15 chickens, another 17, and a third has 14; how many chickens in all?

9. Andrew learned 75 verses of poetry in one week, 94 in another, and 87 in a third; how many verses did he learn in three weeks?

10. John gave a beggar 53 cents, Willie gave him 72 cents, and Andrew gave him 65 cents; how many cents did he receive?

11. A man bought a horse for 95 dollars, a cow for 40 dollars, and a wagon for 65 dollars; how much did he invest?

12. A merchant commenced business with $9,875; the first year his net profits were $2,134, the second year $1,654, the third year $2,547, and the fourth year $2,620; what was then the amount of his funds?

13. A merchant commenced with $12,650; the first year he gained $2,163, the second $1,875, the third $1,260, and the 4th year he lost $4,105; what funds had he left?

14. A man commenced with $4500; the first year he doubled his money, but at the beginning of the next year he lost $2500; he then doubled what remained, and lost $6000; then he doubled what remained and closed business with what amount of money?

15. A man bought 3000 acres of land; he then sold to one man 324 acres, to another 236 acres, to a third 148 acres, to a fourth 465 acres, and to a fifth 634 acres; how many acres were left?

16. A merchant took with him $5000 to buy goods; he purchased dry goods for $1864, groceries for $1256; hardware for $630, hats and boots for $362; how much had he left?

17. A man owns three farms; the first contains 275 acres, the second 483 acres, and the third 1230 acres; he sells the first farm, 236 acres of the second, and 584 acres of the third; how many acres remain unsold? how many of the second farm? and how many of the third?

18. A man bought a horse for $150, and a buggy and harness for $275; he sold them, gaining $42; what did he sell them for?

19. John Jones bought a farm for $6875; he paid for repairs $2172, and then sold it for $10165; how much did he gain or lose by the transaction?

20. James Johnson bought one house for $6,540, another for $7,965, and a third for $12,384; he paid for repairs $3,165, and then sold the three houses for $31,236; how much did he gain or lose by the transaction?

21. Invested 3245 dollars in property, and sold it so as to gain 534 dollars. What was it sold for?

22. A merchant bought dry goods for 576 dollars, groceries for $375, and hardware for $234; in selling he gains $156; what was the amount of sales? *Ans.* $1,341.

23. A, B, and C engage in trade; A puts in $2,576, B $3,845, and C $431 more than A and B; what is the amount of their stock? *Ans.* $13,273.

24. A man is 9 years older than his wife, and she is 22 years older than their son, who is 12 years old; what is the age of each? *Ans.* Wife's age, 34 y.; man's, 43 y.

25. A man bequeathed to his wife $5,650, to each of his three sons $3,775, and to each of his two daughters $3,550; what was the amount of his bequests?

Ans. 24,075.

26. A man owing $9,856, pays at one time $3,453, and at another $2,176; how much does he still owe?

Ans. $4,227.

27. The sum of two numbers is 8,643, and one of the numbers is 5,756; what is the other number?

28. The number of pupils registered is 1,253, and the number at school is 1,168; how many are absent?

29. The sum of two numbers is 6,432, and one of the numbers is 2,541; what is the difference of the numbers?

Ans. 1,350.

30. The sum of two numbers is 5,680, and their difference is 596; what are the numbers?

Ans. 3,138 and 2,542.

31. The difference of two numbers is 265, and the smaller number is 576; what is the larger number?

32. The difference of two numbers is 175, and the larger number is 651; what is the sum of the two numbers? *Ans.* 1,127.

33. Add 32,745, 276, 304,721, 5,640, 216, 243, 976,874, 2,176, 81,275, 9,406, 20,045, 6,320,754, and 7,105,006.
Ans. 14,859,377.

34. Fifteen gallons leaked out of a cask of 81 gallons; how many gallons were left?

35. A merchant engaged in trade with $5,481, and gained each year for five years $1,254; but the sixth year he lost $2,162; what was then the amount of his capital?
Ans. $9,589.

36. A father left an estate of $32,600 to his five children, the eldest to have $1,000 more than the next, and the next $1,000 more than his next younger brother, and so on to the youngest; what was the share of each?
Ans. Eldest, $8,520, 7,520, 6,520, 5,520, 4,520.

37. A man purchased 1,200 acres of land, and sold it in parcels as follows: to one man, 364 acres; to another, 204 acres; to a third, 468 acres, and the balance (how many acres?) to a fourth. *Ans.* 164 acres.

38. A man bought one farm for $6,748, and another for $4,482; he then sold both for $12,000; how much did he gain or lose? *Ans.* $770 gain.

39. A man sold pork for $75; beef for $36; butter for $25, and cheese for $18; he then bought sugar for $12; coffee for $15; salt for $5, and dry goods for $31; how much cash had he left? *Ans.* $91.

40. Columbus discovered America in 1492, and Washington achieved our Independence in 1783; what time elapsed between those two great events?

41. Washington died in 1799, at the age of 67 years; in what year was he born?

42. A merchant purchased goods for $2,154, on which he paid $1,465; how much did he still owe?

MULTIPLICATION.

MULTIPLICATION AND DIVISION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

As a Multiplication Table, begin with the first line; thus,

Once 1 is 1; twice 1 are 2; three times 1 are 3, etc. Second line, Once 2 are 2; twice 2 are 4; 3 times 2 are 6; 4 times 2 are 8, etc. Third line, Once 3 are 3; twice 3 are 6; 3 times 3 are 9; 4 times 3 are 12, etc. Recite each line similarly.

REM. 4 times 3 are 12, and 3 times 4 are 12; hence, alternating the factors does not change the product.

As a Division Table, begin with the first line; thus, 1 is contained in 1, once; in 2, twice; in 3, 3 times; in 4, 4 times, etc. Second line, 2 into 2 = 1; 2 into 4 = 2; 2 into 6 = 3; 2 into 8 = 4, etc. Third line, 3 into 3 = 1; 3 into 6 = 2, etc.

REM.—As a Multiplication Table, it may also be read by the column, by which the factors are alternated, without changing the product. Any number is multiplied by 10 by adding a zero to it. As a Division Table, the first column has all the divisors, the first line all the quotients, and every number in each line is a dividend, which is always in the same line and the same column with the quotient and divisor. Any number having a zero in the units place is divided by 10 by removing the zero.

THEOREM I.

Any number is multiplied by 10 by annexing a zero to it.

Since the product of any number multiplied by 1 is equal to the number itself, the product of any number multiplied by 2 is double the number, etc.

For, as

$$10 \times 1 = 10, \text{ and } 10 \times 2 = 20, \text{ and } 10 \times 24 = 240,$$

and as alternating the factors does not change the product, hence,

$$1 \times 10 = 10, \text{ and } 2 \times 10 = 20, \text{ and } 24 \times 10 = 240.$$

∴ Any number is multiplied by 10 by annexing a zero to it.

COR.—Any number is multiplied by 100 by annexing two zeros to it, and annexing three zeros multiplies it by 1000, etc.

THEOREM II.

The product of any two factors will have as many figures, or one less, than both factors.

1	3	3	4	9	50	500	500
1	3	4	4	9	5	5	50
1	9	12	16	81	250	2500	25000

The products of the smaller figures of units will be but one figure until above 3, when there will be two figures, but never more, as 9 × 9 = 81, and every additional figure annexed to each or either factor, whether small or large, will make an increase of one figure and no more; therefore the product of any two factors will have as many figures, or one less than both factors.

Cor. 1.—The product of any two figures cannot be less than one figure, nor more than two.

Cor. 2.—The product of units by units must be units, and when there are two figures, the left-hand figure will be tens. The product of tens by units must be tens, and when there are two figures, the left-hand figure will be hundreds; and if any order be multiplied by units, the right-hand figure of the product will be the same order as the multiplicand, and if there be two figures in the product, the left-hand figure will belong to the next higher order.

Cor. 3.—When the multiplier is tens, the product will be ten times as great as if the multiplier were units; that is, each product will have one zero to the right of it, holding the units place, or the first figure of the product must be placed in the column of tens; when the multiplier is hundreds, the right-hand figure must be placed in the column of hundreds; and, in general, whatever the order of the multiplier is, the right-hand figure must be in the column of that order.

COR. 4.—If there be one or more zeros in the multiplier, the product of the next figure will be put back one figure for every zero.

REM.—In the multiplication, each figure may be regarded as the unit of its order.

PROBLEMS.

1. $10 \times 10 = 100.$

2. $$\begin{array}{r} 11 \times 11 = 121 = 11 \times (10 + 1) = 11 \times 1 = 11 \\ 11 \times 10 = \underline{110} \\ 121 \end{array}$$

3. $$\begin{array}{r} 12 \times 12 = 144 = 12 \times (10 + 2) = 12 \times 2 = 24 \\ 12 \times 10 = \underline{120} \\ 144 \end{array}$$

4. Multiply 432 by 4 $= (400 + 30 + 2) \times 4.$

$$\begin{array}{rrrcr} \therefore & 2 \times 4 = & 8 & \text{and} & 432 \\ & 30 \times 4 = & 120 & & \underline{4} \\ & 400 \times 4 = & \underline{1600} & & 1728 \\ & & 1728 & & \end{array}$$

5. Multiply 432 by 14 $= 432 \times (10 + 4).$

$$\begin{array}{rrrcr} \therefore & 432 \times 4 = & 1728 & \text{or} & 432 \\ & 432 \times 10 = & \underline{4320} & & \underline{14} \\ & & 6048 & & 1728 \\ & & & & \underline{432} \\ & & & & 6048 \end{array}$$

REM.—The problems should be carefully impressed on the mind before proceeding.

6.

$$\begin{array}{r} 432 \\ \underline{124} \\ 1728 \\ 864 \\ \underline{432} \\ 53568 \end{array} \qquad \begin{array}{rcr} 432 \times 4 &=& 1728 \\ 432 \times 20 &=& 8640 \\ 432 \times 100 &=& \underline{43200} \\ && 53568 \end{array}$$

Cor. 1.—When the multiplicand has several figures and the multiplier one that is only units, the first product of units by units will be units, or units and tens; the units must be placed in the right-hand or units place; if there be tens, it must be reserved and placed in or added to the column of tens; in the next product of tens by units, the right-hand figure will be tens, and must be united with the tens reserved, and placed in the column of tens; the left-hand figure, if there be one, must be treated as the previous one, reserved until the next product is obtained, and united with the right-hand figure; the process is the same in every successive order.

Cor. 2.—When the multiplier also has several figures, the process of each successive multiplier is the same, except that the right-hand figure of each product must be placed in the order of its multiplier. (Cor. 3, Prob. 2, page 22.)

Rem.—A multiplicand may be either an abstract or a concrete number, but a multiplier cannot be concrete, as it cannot refer to things, but merely indicates how many times the multiplicand is to be taken; but the product will be of the same name as the multiplicand; for twice \$5 are \$10; 3 times 20 yards of cloth are 60 yards of cloth; twice 4 are 8; 3 times 4 are 12, etc.

In computation, it is best to regard all numbers as abstract.

(7.)
```
   36425
     324
   ------
  145700
  72850
109275
--------
11801700
```

(8.)
```
  26432
    104
  ------
 105728
26432
-------
2748928
```

(9.)
```
   26432
    3004
   ------
  105728
79296
--------
79401728
```

(10.)
```
     26432
     50004
     ------
    105728
132160
----------
1321705728
```

(11.)
```
  234
  123
  ---
  702
 468
234
-----
28782
```

(12.)
```
  123
  234
  ---
  492
 369
246
-----
28782
```

Rem.—The product is not changed by alternating the multiplicand and multiplier.

EXAMPLES.

1. Multiply 54326 by 346.
2. Multiply 23748 by 543.
3. Multiply 46874 by 697.
4. Multiply 36975 by 476.
5. Multiply 236874 by 2134.
6. Multiply 9876325 by 356.
7. Multiply 879654 by 2175.
8. Multiply 986432 by 8704.
9. Multiply 326875 by 3005.
10. Multiply 468753 by 2100.

Examples may be added, or the same repeated, as the student will more readily comprehend by repetition than by different examples.

Rem. 1.—In multiplication, two factors are given to find their product.

Rem. 2.—In division, two numbers also are given to find the third; the one called the dividend corresponds to the product in multiplication, the other given number is called the divisor, and the required number is called the quotient; the two latter correspond to the factors in multiplication.

DIVISION.

PROBLEMS.

When the product of two numbers is 4, and one of the numbers is 2, the other number is also 2; for $2 \times 2 = 4$, and 4 divided by 2, or 4 divided into 2 equal parts, each part is 2, that is, the quotient is 2.

1. $9 \div 3 = 3$.
2. $12 \div 2 = 6$.
3. $12 \div 3 = 4$.
4. $16 \div 4 = 4$.
5. $15 \div 3 = 5$.
6. $15 \div 5 = 3$.

COR. 1.—The product of the divisor and quotient equals the dividend.

COR. 2.—The divisor and quotient may be alternated.

$$\begin{array}{r} 24 \\ \underline{6} \\ 18 \\ \underline{6} \\ 12 \\ \underline{6} \\ 6 \\ \underline{6} \\ 0 \end{array}$$

COR. 3.—Division is the reverse of multiplication and addition, and is similar to subtraction; for, it is separating a number into equal parts, which is the same as subtracting the same number from a larger one; that is, subtracting the divisor from the dividend and then from the remainder, repeating this process until there is no remainder, or until the remainder is less than the divisor. 6 is subtracted 4 times, hence it is contained four times. $24 \div 6 = 4$.

(1.)

$$\begin{array}{l} 10\;)\;100\;(\;10 \\ \underline{10} \\ 0 \end{array}$$

(2.)

$$\begin{array}{l} 11\;)\;121\;(\;11 \\ \underline{11} \\ 11 \\ \underline{11} \end{array}$$

(3.)

$$\begin{array}{l} 12\;)\;144\;(\;12 \\ \underline{12} \\ 24 \\ \underline{24} \end{array}$$

(4.)

$$\begin{array}{l} 11\;)\;121\;(\;10+1 \\ \underline{110} \\ 11 \\ \underline{11} \end{array}$$

(5.)

$$\begin{array}{l} 12\;)\;144\;(\;10+2 \\ \underline{120} \\ 24 \\ \underline{24} \end{array}$$

6. $48 \div 12 = 4.$
7. $64 \div 8 = 8.$
8. $96 \div 12 = 8.$
9. $12 \times 4 = 48.$
10. $8 \times 8 = 64.$
11. $12 \times 8 = 96.$
12. $120 \div 10 = 12.$
13. $130 \div 10 = 13.$
14. $140 \div 10 = 14.$
15. $10 \times 12 = 120.$
16. $10 \times 13 = 130.$
17. $10 \times 14 = 140.$

Cor. 1.—Adding a zero to the right of a number multiplies the number by 10; taking a zero away from the right of a number divides the number by 10.

Divide 60536 by 4; thus,

$$\begin{array}{rr} 4\;)\;60536\;(& 10000 \\ \underline{40000} & \\ 20536\;(& 5000 \\ \underline{20000} & \\ 536\;(& 100 \\ \underline{400} & \\ 136\;(& 30 \\ \underline{120} & \\ 16\;(& \underline{4} \\ \underline{16} & 15134 \end{array}$$

or

$$\begin{array}{l} 4\;\underline{)\;60536} \\ 15134 \end{array}$$

The divisor 4 is contained once in the unit of the highest order of the dividend, which is one ten-thousand; into the remainder 5000 times, then 100, 30 and lastly 4.

Rem. 1.—The same result is obtained by short division, by putting the first figure of the quotient under the left-hand figure of the dividend (when it is contained in it), as it is of the same order.

Rem. 2.—If the unit of the divisor is not contained in the first unit of the dividend, then the first figure of the quotient will be of the same order as the second figure of the dividend and should be placed under it.

Divide 60536 by 14; thus,

$$
\begin{array}{lcl}
14\ )\ 60536\ (\ 4324 & \text{and} & 214\ )\ 925336\ (\ 4324 \\
\underline{56} & & \underline{856} \\
45 & & 693 \\
\underline{42} & & \underline{642} \\
33 & & 513 \\
\underline{28} & & \underline{428} \\
56 & & 856 \\
\underline{56} & & \underline{856} \\
4324 \times 14 = 60536. & & 4324 \times 214 = 925336.
\end{array}
$$

Cor. 1.—Since the product of any two factors will have as many figures or one less than both factors, so in division the number of figures of the divisor and quotient will either be equal to or one greater than that of the dividend.

Cor. 2.—When the divisor is contained in the same number of figures of the dividend as is in the divisor, then the number of figures of the divisor and quotient will be one more than that of the dividend; but when it requires an additional figure of the dividend to contain the divisor, then the number of figures of the divisor and quotient will be equal to that of the dividend.

11. What cost 245 lbs. of beef at 8 cts. per lb.?

12. At 8 cts. per lb., how many lbs. of beef can be bought for 1960 cts.?

13. What cost 348 acres of land at $45 per acre?

14. At $48 per acre, how many acres can be bought for $15660?

15. What cost 3245 acres of land at $64 per acre?

16. At $64 per acre, how many acres can be bought for $207680?

17. What is the cost of 15 horses at $125 each?

18. How many horses at $125 each can be bought for $1875?

19. What is the cost of 35 oxen at $75 each?

20. How many oxen at $75 each can be bought for $2625?

21. What is the cost of 84 cows at $45 each?

22. How many cows at $45 each can be bought for $3780?

23. Multiply 54682 by 9; thus,

$$\begin{array}{rcrcr} 54682 & \times & 10 & = & 546820 \\ 54682 & \times & 1 & = & \underline{54682} \\ & & & & 492138 \end{array}$$

24. Multiply 54682 by 99.

$$\begin{array}{rcrcr} 54682 & \times & 100 & = & 5468200 \\ 54682 & \times & 1 & = & \underline{54682} \\ & & & & 5413518 \end{array}$$

25. Multiply 54682 by 999.

$$\begin{array}{rcrcr} 54682 & \times & 1000 & = & 54682000 \\ 54682 & \times & 1 & = & \underline{54682} \\ & & & & 54627318 \end{array}$$

26. Multiply 54682 by 25. Instead of 25, multiply by $\frac{100}{4}$.

$$\begin{array}{r} 4\,\underline{)\,5468200} \\ 1367050 \end{array}$$

27. Multiply 54682 by 50. Instead of 50, multiply by $\frac{100}{2}$.

28. Multiply 54682 by 75. Multiply the result ot 26th by 3.

29. Multiply 54682 by 150.

$$\begin{array}{lcr} 54682 \times 100 & = & 5468200 \\ \text{Adding } \frac{1}{2}, & & \underline{2734100} \\ & & 8202300 \end{array}$$

30. Divide 546825 by 25. Multiply by 4 and strike off two figures.

31. Divide 546850 by 50.

$$\begin{array}{r} 5468|50 \\ \underline{2} \\ 10937.00 \end{array}$$

Rem.—As a number is multiplied by 100 by adding two zeros, so any number is divided by 100 by cancelling the two right-hand figures, which will then form the remainder.

32. Bought 24 horses at $84 each, 54 cows at $36 each, and 364 sheep at $4 each; what did the horses cost? the cows? the sheep? what did all cost?

33. Sold all the stock of the last example at a profit of $324; what did I sell them for?

34. Bought 3064 acres of land at $25 per acre, and sold it at a loss of $1245. What did the land cost? and what was it sold for?

35. Bought 3840 acres of land at $10 per acre; divided it into twenty farms of an equal number of

acres each; twelve of the farms I sold at $15 per acre, 3 of the farms at $12 per acre, and the five remaining farms at $4 per acre; did I gain or lose, and how much?

36. A farm of 364 acres was bought for $9100 and sold at a gain of $1456; what was paid per acre? and at what rate was it sold?

37. Bought 184 acres of land for $11960, and sold it for $13064; what was the cost per acre? and at what rate was it sold?

38. The cost of 12 horses and 15 cows was $2760; the horses cost $180 each; what was the average cost of each cow?

39. In a square mile there are 640 acres; how many farms of 160 acres each in a State that has 9,000 square miles? how many in one of 46,000 sq. miles? how many in one of 257,000 sq. miles?

40. What is the value of the land in the first State at $25 per acre? in the second at $20 per acre? and in the third at $5 per acre?

41. A man having an estate worth $15,000, increases it $2500 every year for twenty-five years, when he dies, leaving it as follows: To his wife $20,000, to his eldest son $8000, to his second son $7000, to his third son $7000, and the remainder to be divided equally among his five daughters; what is the share of each daughter?

42. A merchant commences business with a capital of $25000; at the end of the first year he finds that he has increased it $5000, the second year the increase is $4500, and the same at the end of the third year, when he transfers the whole business to his three sons. What is the capital of each son? *Ans.* $13000.

REVIEW.

NUMERATION.

Write in figures the following numbers:

Twenty-five, One hundred and twenty-five, Two hundred and five, Three hundred and six, Four hundred and eight, One thousand, One thousand three hundred and forty-five, Five thousand eight hundred and four, Ten thousand five hundred and nine, Thirty-four thousand and twenty, Fifty-three thousand and five, Five hundred thousand, Six hundred thousand and four, One million, One million four hundred and twenty-five thousand six hundred and twelve.

ADDITION.

ORAL QUESTIONS.

John has 8 apples, Willie has 7, and Andrew 5; how many apples have the three boys?

Bought coal for $8, wood for $6, and a barrel of flour for $5; what was the amount of the purchase?

How many are

6+4+5
8+7+4
3+8+7
4+6+9
2+5+8
5+9+2
1+3+6
9+6+4
8+7−5
9+8−7
7+9+8
6+5+4−6
5+6+8−5
4+5+9−8
3+9+7−9
2+7+8−4

How many are 3+7; 13+7; 23+7; 43+7; 53+7; 63+7; 73+7; 83+7; 93+7?

How many are 3+8; 13+8; 23+8; 33+8; 43+8; 53+8; 63+8; 73+8; 83+8; 93+8?

How many are 4+7; 14+7; 24+7; 34+7; 44+7; 54+7; 64+7; 74+7; 84+7; 94+7?

How many are 5+7; 15+7; 25+7; 35+7; 45+7; 55+7; 65+7; 75+7; 85+7; 95+7?

How many are 5+8; 15+8; 25+8; 35+8; 45+8; 55+8; 65+8; 75+8; 85+8; 95+8?

How many are 5+9; 15+9; 25+9; 35+9; 45+9; 55+9; 65+9; 75+9; 85+9; 95+9?

How many are 6+7; 16+7; 26+7; 36+7; 46+7; 56+7; 66+7; 76+7; 86+7; 96+7?

How many are 6+8; 16+8; 26+8; 36+8; 46+8; 56+8; 66+8; 76+8; 86+8; 96+8?

How many are 7+7; 17+7; 27+7; 37+7; 47+7; 57+7; 67+7; 77+7; 87+7; 97+7?

How many are 7+8; 17+8 : 27+8; 37+8; 47+8; 57+8; 67+8; 77+8; 87+8; 97+8?

These questions should be frequently repeated or similar ones asked.

EXAMPLES.

Add the following numbers:

1. 1	2. 10	3. 100	4. 111
2	20	200	222
3	30	300	333
4	40	400	444
5	50	500	555
6	60	600	666
7	70	700	777
8	80	800	888
9	90	900	999
—	—	—	—

After adding the first three examples, add their sums, and observe the correspondence with Ex. 4, when the tens and hundreds are added with the columns of tens and hundreds.

5.	6.	7.	8.	9.	10.
28	53	45	56	79	84
36	24	61	43	86	73

11.	12.	13.	14.	15.	15.
121	312	276	751	542	374
343	256	543	324	324	236
621	461	631	263	261	573

17. What is the sum of 4,321, 604, and 36?
18. What is the sum of 264, 301, and 5?
19. What is the sum of 205, 25, and 6?
20. What is the sum of 4, 24, 324?
21. What is the sum of 3,276, 2,132, and 5,437?
22. What is the sum of 57,896, 98,765, and 78,901?

Similar examples and *larger* ones may be given for the slate or blackboard.

SUBTRACTION.

ORAL QUESTIONS.

Take 7 from 9; 7 from 19; 7 from 29; 7 from 39; 7 from 49; 7 from 59; 7 from 69; 7 from 79; 7 from 89; 7 from 99; 7 from 109.

Take 6 from 13; 6 from 23; 6 from 33; 6 from 43; 6 from 53; 6 from 63; 6 from 73; 6 from 83; 6 from 93; 6 from 103.

Take 8 from 14; 8 from 24; 8 from 34; 8 from 44; 8 from 54; 8 from 64; 8 from 74; 8 from 84; 8 from 94; 8 from 104.

Take 9 from 16; 9 from 26; 9 from 36; 9 from 46; 9 from 56; 9 from 66; 9 from 76; 9 from 86; 9 from 96; 9 from 106.

Take 2 from 11; 2 from 21; 2 from 31; 2 from 41; 2 51; 2 from 61; 2 from 71; 2 from 81; 2 from 91; 2 from 101.

Take 3 from 12; 3 from 22; 3 from 32; 3 from 42; 3 from 52; 3 from 62; 3 from 72; 3 from 82; 3 from 92; 3 from 102.

Take 4 from 13; 4 from 23; 4 from 33; 4 from 43; 4 from 53; 4 from 63; 4 from 73; 4 from 83; 4 from 93; 4 from 103.

Take 5 from 14; 5 from 24; 5 from 34; 5 from 44; 5 from 54; 5 from 64; 5 from 74; 5 from 84; 5 from 94; 5 from 104.

Take 2 from 10; 2 from 20; 2 from 30; 2 from 40; 2 from 50; 2 from 60; 2 from 70; 2 from 80; 2 from 90; 2 from 100.

These questions should be often repeated.

EXAMPLES.

1. From 10 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
2. From 20 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
3. From 30 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
4. From 40 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
5. From 50 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
6. From 60 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
7. From 70 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
8. From 80 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
9. From 90 take 1; 2, 3, 4, 5, 6, 7, 8, 9.
10. From 100 take 1; 2, 3, 4, 5, 6, 7, 8, 9, 99.
11. From 200 take 1; 2, 3, 4, 5, 6, 7, 8, 9, 199.
12. From 300 take 1; 2, 3, 4, 5, 6, 7, 8, 9, 299.
13. From 400 take 1; 2, 3, 4, 5, 6, 7, 8, 9, 399.

14. From 1,000 take 1; 2, 3, 4, 5, 6, 7, 8, 9, 999.
15. From 10,000 take 1; 2, 3, 4, 5, 6, 7, 8, 9, 9,999.
16. From 100,000 take 99,999.
17. From 2,000,000 take 1,999,999.
18. From 3,246,532 take 2,164,320.
19. From 800 take 400.
20. From 8,000 take 4,000.
21. From 545 take 116.
22. From 576 take 144.
23. From 624 take 304.
24. From 5,324 take 1,304.
25. From 6,725 take 2,432.
26. From 74,832 take 68,741.
27. From 83,754 take 48,623.

28. Columbus discovered America in the year 1492; how many years since?

29. Washington was born in 1732, and lived 67 years; in what year did he die?

30. I owed $1,000, and I paid one man $324, another $204, a third $120, and a fourth $67; how much do I still owe? *Ans.* $285.

31. A man dying leaves $5,000 to his widow and son, the widow to have $1,000 more than the son; what is the portion of each? *Ans.* Son, $2,000; widow, $3,000.

32. A father divides his estate of $12,000 as follows: To each of his two daughters, $2,000; to one son, $1,500; to another, $2,000; to a third son, $2,500, and the balance to the widow; what is her share? *Ans.* $2,000.

ADDITION AND SUBTRACTION.

EXAMPLES.

1. A man has six farms; in the 1st 6,312 acres; in the 2d 3,241 acres; in the 3d 4,276 acres; 4th, 272 acres; 5th, 304 acres, and in the 6th, 63 acres; how many acres in all? *Ans.* 14,468 acres.

2. A merchant sold goods for $6,032, gaining thereby $326. What did the goods cost? *Ans.* $5,706.

3. A man owning 2,142 acres of land, gave to his eldest son 834 acres, to his second son 612 acres, and to the third 420 acres; how many acres were left? *Ans.* 276 acres.

4. A man left real estate worth $9,376, and personal property worth $2,142; he owes one man $1,236, another $875, and a third $450; what is the net value of his estate? *Ans.* $8,957.

5. A, B, and C commenced business; A put in $3,275, B $4,150, and C $5,180; when they closed they found that they had lost $1,200; what had they at the beginning and at the close?

Ans. Commenced with $12,605 and ended $11,405.

6. A horse and buggy together are worth $500; but the horse is worth $100 more than the buggy; what is the value of each? *Ans.* Horse, $300; buggy, $200.

7. The sum of two numbers is 5,439, and the one number is 215 greater than the other; what are the numbers?

8. The minuend is 5,746, and the subtrahend 3,825; what is the difference?

9. The subtrahend is 3,825, and the difference 1,921; what is the minuend?

10. The minuend is 5,746 and the difference 1,921; what is the subtrahend?

11. When you know the minuend and difference, how do you find the subtrahend?

12. How do you find the minuend, knowing the subtrahend and difference?

MULTIPLICATION.

ORAL EXERCISES.

If an apple is worth 2 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 3 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 4 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 5 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 6 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 7 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 8 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 9 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 10 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 11 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

If an apple is worth 12 cents, what are 2 apples worth? 3? 4? 5? 6? 7? 8? 9? 10? 11? 12?

EXAMPLES.

1. Multiply 13 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
2. Multiply 14 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
3. Multiply 15 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
4. Multiply 16 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
5. Multiply 17 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
6. Multiply 18 by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
7. Multiply 256 by 4, 24, 124.
8. Multiply 2,145 by 4, 24, 124.
9. Multiply 3,636 by 5, 20, 100, 125.
10. Multiply 6,435 by 5, 20, 300, 1,000, and by 1,325. Analyze.
11. Multiply 3,674 by 104, 206, 3,005.
Ans. 382,096, 756,844, and 11,040,370.
12. Multiply 21,564 by 2,100, 2,010, 2,001.
Ans. 45,284,400, 43,343,640, and 43,149,564.
13. Multiply 54,000 by 1,200, 14,000, 630.
Ans. 64,800,000, 756,000,000, and 34,020,000.
14. Multiply 60,401 by 1,040, 1,004, 1,400.
Ans. 62,817,040, 60,642,604, and 84,561,400.
15. Multiply four thousand and three by three hundred and one. *Ans.* 1,204,903.
16. Multiply forty-six thousand one hundred and three by three thousand and two. *Ans.* 138,401,206.
17. Multiply four hundred and fifty-six thousand three hundred and fifty-four by three thousand five hundred and thirty-six. *Ans.* 1,613,667,744.
18. Multiply four millions three hundred and fifty-four thousand six hundred and twenty-five by thirty-six thousand four hundred and sixty-seven.
Ans. 158,800,109,875.

19. Multiply sixty-four millions seven hundred and ninety-five thousand eight hundred and sixty-nine by six hundred and fifty-four thousand three hundred and seventy-eight. *Ans.* 42,400,991,164,482.

20. Multiply six hundred and four millions two hundred thousand four hundred and four by four hundred and five thousand two hundred.

Ans. 244,822,003,700,800.

21. What is the product of 3,754 and 268.

Ans. 1,006,072.

22. The factors of a number are 57 and 29; what is the number? *Ans.* 1,653.

23. Two of the three factors of the number 1,188 are 9 and 12; what is the third factor? *Ans.* 11.

DIVISION.

ORAL EXERCISES.

If one apple cost 2 cents, how many can you buy for 4 cents? 6? 8? 10? 12?

How often is 2 contained in 4? 6? 8? 10? 12? 14? 16? 18? 20? 22? 24?

How often is 3 contained in 6? 9? 12? 15? 18? 21? 24? 27? 30? 33? 36?

How often is 4 contained in 8? 12? 16? 20? 24? 28? 32? 36? 40? 44? 48?

How often is 5 contained in 10? 15? 20? 25? 30? 35? 40? 45? 50? 55? 60?

How often is 6 contained in 12? 18? 24? 30? 36? 42? 48? 54? 60? 66? 72?

How often is 7 contained in 14? 21? 28? 35? 42? 49? 56? 63? 70? 77? 84?

How often is 8 contained in 16? 24? 32? 40? 48? 56? 64? 72? 80? 88? 96?

How often is 9 contained in 18? 27? 36? 45? 54? 63? 72? 81? 90? 99? 108?

How often is 10 contained in 20? 30? 40? 50? 60? 70? 80? 90? 100? 110? 120?

How often is 11 contained in 22? 33? 44? 55? 66? 77? 88? 99? 110? 121? 132?

How often is 12 contained in 24? 36? 48? 60? 72? 84? 96? 108? 120? 132? 144?

Divide 12 into 2 equal parts; into 3, 4, 6.

Divide 24 into 2 equal parts; into 3, 4, 6.

Divide 36 into 2 equal parts; into 3, 4, 6.

Divide 48 into 2 equal parts; into 3, 4, 6.

If 10 yards of cloth cost $50, what is the cost of one yard?

If 4 pecks make one bushel, how many bushels in 36 pecks?

If 48 bushels of wheat cost $48, what is the cost of one bushel?

If 24 bushels of wheat cost $48, what is the cost of one bushel?

Divide 48 apples equally among 4 boys; 6 boys; 8, 12.

MULTIPLICATION AND DIVISION.

EXAMPLES.

1. At $5 each, how many sheep can be bought for $25? $50? $100? $200? $300? $400? $500? $600? $625? $650? $675?

2. At $6 each, how many for $24? $48? $96? $120? $132? $144? $240? $264? $300? $336? $348?

3. Divide $1,200 into 5 equal parts; how many in each part?

4. Divide 48,000 into 4 parts, or two equal pairs, so that each number of the first pair shall be one-half of one of the numbers of the second pair.

Ans. Each of the first pair 8,000, and each of the second 16,000.

5. Divide 24,000 into three parts, so that the difference of any two consecutive parts shall be 1,000.

Ans. 7,000, 8,000, and 9,000.

6. How long would it take a man to travel around the earth, whose circumference is about 25,000 miles, if he travel 50 miles a day?

7. A man purchased the same number of horses and cows for $5,640, the horses at $85 each, and the cows at $35 each. How many of each kind? *Ans.* 47 of each.

8. A drover bought sheep, hogs, and cows, of each an equal number, for $2,752, the sheep at $4 each, the hogs at $11, and the cows at $28 each; how many of each kind? *Ans.* 64 of each.

9. The divisor is 463 and the quotient 264; what is the dividend? *Ans.* 122,232.

10. The quotient is 327 the dividend 160,557; what is the divisor? *Ans.* 491.

11. A farmer sold 8 cows and 12 horses for $1,092; the horses were rated at $75 each; what were the cows sold at? *Ans.* $24.

12. A man bought a farm of 225 acres for $10,125, and sold it for $11,250; what did he pay per acre, and how much per acre did he gain?

Ans. He paid $45 and gained $5 per acre.

13. A clerk has a salary of $80 per month; he pays for board $25 per month and other expenses $15 per month; how much does he save per month? how much per year?

Ans. $40 per month and $480 per year.

14. If I pay $225 for 75 sheep, and they cost $1 each for keeping, and I sell them at $5 each, how much is my profit? *Ans.* $75.

15. If 8 cords of wood cost $32, what is the cost of 1 cord? of 2 cords? 3? 4? 5? 6? 7? 8? 9?

16. If 9 lbs. sugar cost 63 cents, what is the cost of 11 lbs.?

17. If 12 lbs. coffee cost 300 cents, what cost 5 lbs.?

18. If 8 lbs. wool cost 280 cents, what cost 15 lbs.?

Ans. 525 cents.

19. A farmer sold 235 barrels of flour at $6 per barrel; he bought goods for $830, and for the balance took coal at $4 per ton; how many tons did he get? *Ans.* 145 tons.

20. The quotient is 164, the divisor 235, and the remainder 143; what is the dividend? *Ans.* 38,683.

21. How many revolutions will a wheel 20 feet in circumference make in running a mile (5,280 ft.)?

Ans. 264.

22. If two men can do a piece of work in 3 days, how long will it take one man to do it? *Ans.* 6 days.

23. If 4 men can do a piece of work in 6 days, how long will it take 3 men to do it? *Ans.* 8 days.

24. If 15 acres of land cost $375, what is the cost of 1 acre? 2 acres? 3 acres? 4 acres? 5 acres? 55 acres?

Ans. $25, $50, $75, $100, $125, $1,375.

25. If 5 tons of hay cost $60, what is the cost of 1 ton? 2 tons? 3 tons? 45 tons? 75 tons?

Ans. $12, $24, $36, $540, $900.

26. If 36 acres of land cost $1,440, what will 57 acres cost? 84 acres? *Ans.* $2,280 and $3,360.

27. Sold 12 horses at $75 each and 15 cows at $24 each, and invested the proceeds in sheep at $5 each; how many sheep did I get? *Ans.* 252 sheep.

28. Sold 275 acres of land at $24 per acre, 340 acres at $32 per acre, and invested the proceeds in government land at $2 per acre; how many acres did I get.

Ans. 8,740 acres.

29. Of what number is 36 both divisor and quotient?

Ans. 1,296.

30. Bought 12 horses at $80 per head; at what price must I sell each horse so as to gain $72. *Ans.* $86.

31. A party of 45 men has provisions sufficient for 40 days; how long would it last 1 man? 2 men? 4 men? 5 men? 20 men?

Ans. 1 man, 1,800 days; 2 men, 900 days; 4 men, 450 days; 5 men, 360 days; 20 men, 90 days.

32. A garrison of 900 men have provisions for 6 months; how many men would the same provisions last 9 months?

Ans. 600 men.

33. If 14 men do a piece of work in 3 days, how many men would do it in 1 day? how many in 2 days? in 6 days?

Ans. 42 men in 1 day, 21 men in 2 days, 7 men in 6 days.

34. If 4 men mow a field of grass in 3 days, how long will it take 1 man? 2 men? 3 men? 6 men?

Ans. 1 man, 12 days; 2 men, 6 days; 3 men, 4 days; 6 men, 2 days.

FACTORING.

Any number that is the product of two or more numbers is a ***Composite Number;*** as, 12 is the product of 4 and 3, and 4 is the product of 2 and 2; hence, the factors of 12 are 2, 2, and 3; these factors cannot be reduced, they are therefore called ***Prime Factors,*** as any number is called Prime which is not formed by other factors than itself and unity. There are sixteen Prime Numbers in the first fifty; viz., 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47, and they are found thus:

1, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~;
11, ~~12~~, 13, ~~14~~, ~~15~~, ~~16~~, 17, ~~18~~, 19, ~~20~~,
~~21~~, ~~22~~, 23, ~~24~~, ~~25~~, ~~26~~, ~~27~~, ~~28~~, 29, ~~30~~,
31, ~~32~~, ~~33~~, ~~34~~, ~~35~~, ~~36~~, 37, ~~38~~, ~~39~~, ~~40~~,
41, ~~42~~, 43, ~~44~~, ~~45~~, ~~46~~, 47, ~~48~~, ~~49~~, ~~50~~.

Every even number after 2 is composite, as it is divisible by 2; strike these. Every third number after 3 is divisible by 3; strike these. Every fifth number after 5; every seventh number after 7; the ninth numbers are canceled by 3; every eleventh number after 11, etc.

COR. 1.—Every even number is divisible by 2.

COR. 2.—Every number whose last two figures express a number which is a multiple of 4, is divisible by 4; for if the number expressed by these two figures is subtracted from the whole number, the remainder will be a certain number of hundreds which is divisible by 4.

COR. 3.—Every number ending in 5 is divisible by 5.

COR. 4.—Every number ending in zero is divisible by 10, consequently by 2 and 5.

COR. 5.—Every number is divisible by 3, when the sum of its figures taken as units is divisible by 3; for if from 1000 one be subtracted, the remainder is divisible by 9; if from 100 one be subtracted, the remainder is divisible by 9; so also of 10; hence if from 2000 two be subtracted, the remainder is divisible by 9; so also take 2 from 200, and 2 from 20, etc.; therefore, in dividing any number of thousands, hundreds, or tens, the remainder will always be the unit of thousands, hundreds, or tens, and if the sum of these as units and also of the units of the given number equals 9, or any number of nines, the whole number is divisible by 9, and consequently by 3.

PROBLEMS.

Resolve the following numbers to their prime factors:

1. 4 = 2, 2.
2. 6 = 2, 3.
3. 8 = 2, 2, 2.
4. 9 = 3, 3.
5. 10 = 2, 5.
6. 12 = 2, 2, 3.
7. 14 = 2, 7.
8. 15 = 3, 5.
9. 16 = 2, 2, 2, 2.
10. 18 = 2, 3, 3.
11. 20 = 2, 2, 5.
12. 21 = 3, 7.
13. 22 = 2, 11.
14. 24 = 2, 2, 2, 3.
15. 26 = 2, 13.
16. 27 = 3, 3, 3.
17. 28 = 2, 2, 7.
18. 30 = 2, 3, 5.
19. 32 = 2, 2, 2, 2, 2.
20. 33 = 3, 11.
21. 34 = 2, 17.
22. 35 = 5, 7.
23. 36 = 2, 2, 3, 3.
24. 38 = 2, 19.

EXAMPLES.

Resolve the following numbers into their prime factors:

1. 60.
2. 64.
3. 65.
4. 70.
5. 72.
6. 75.
7. 78.
8. 80.
9. 84.
10. 86.
11. 88.
12. 90.
13. 95.
14. 96.
15. 98.
16. 100.
17. 102.
18. 104.
19. 106.
20. 108.
21. 110.
22. 112.
23. 114.
24. 116.
25. 118.
26. 120.
27. 124.
28. 125.
29. 128.
30. 130.
31. 132.
32. 136.
33. 140.
34. 225.
35. 500.
36. 625.

LEAST COMMON MULTIPLE.

DEF.—One number is called a ***Multiple*** of another when it is exactly divisible by that other number.

When a number is resolved into its prime factors, the original number is a multiple of all the prime factors, and of all the quotients arising from these factors divided into the original number; when the same factor occurs several times, the different products of this factor multiplied by itself must also be divided into the original number.

PROBLEMS.

Find the numbers of which the following are multiples:

1. 24. *Ans.* ~~2~~, ~~2~~, 2, 3, 4, 6, 8, 12.
2. 25. *Ans.* 5, 5.
3. 30. *Ans.* 2, 3, 5, 6, 10, 15.
4. 32. *Ans.* ~~2~~, ~~2~~, ~~2~~, ~~2~~, 2, 4, 8, 16.
5. 36. *Ans.* ~~2~~, 2, ~~3~~, 3, 4, 6, 9, 12, 18.

6. 40. *Ans.* ~~2~~, ~~2~~, 2, 5, 4, 8, 10, 20.
7. 48. *Ans.* ~~2~~, ~~2~~, ~~2~~, 2, 3, 4, 6, 8, 12, 16, 24.
8. 54. *Ans.* 2, 3, ~~3~~, ~~3~~, 6, 9, 18, 27.
9. 56. *Ans.* ~~2~~, ~~2~~, 2, 7, 4, 8, 14, 28.

Rem.—First find the prime factors, then the quotients arising by dividing each prime factor and the different products of these factors into the original number.

PROBLEMS.

1. Find the least common multiple of 8 and 12.

8 and 12
2, 2, 2. 2, 2, 3.
$2 \times 2 \times 2 \times 3 = 24$.

It is evident that any number which contains all the prime factors of each number is a common multiple of the given numbers; and the least common multiple must contain these factors and no others.

Cor.—Any factor must enter the L. C. M. as often as it does any given number.

2. Find the least common multiple of 6 and 15.

6 and 15
2, 3. 3, 5.

As 3 is common to both numbers, it must be taken but once. The L. C. M. is $2 \times 3 \times 5 = 30$.

3. Find the least common multiple of 6 and 12.

6 and 12
2, 3. 2, 2, 3.

As 12 is a multiple of 6, it is evident that it contains all the prime factors of 6; hence, whenever one of the given numbers is a multiple of another given number, that other number need not be considered, and may be canceled.

EXAMPLES.

1. Find the least common multiple of 6, 15, and 42.

$$\begin{array}{ccccc} \cancel{6} & \text{and} & 15 & \text{and} & 42 \\ & & 3,\ 5. & & 2,\ 3,\ 7. \end{array}$$

As 42 is a multiple of 6, the 6 may be canceled. Only 3 is common.

$$2 \times 3 \times 5 \times 7 = 210 = \text{L. C. M.}$$

2. Find the least common multiple of 6, 8, 10, 12, 15.

$$\begin{array}{r|rrrrr} 2 & \cancel{6} & 8 & 10 & 12 & 15 \\ \hline 2 & & 4 & \cancel{5} & 6 & 15 \\ \hline & & 2 & & \cancel{3} & 15 \end{array}$$

$$2 \times 2 \times 2 \times 15 = 120$$

As 12 is a multiple of 6, cancel the 6; and as 2 is a common factor of two or more numbers, divide the 2 into them, reserving the divisor, the quotients, and the numbers not divided; as 15 is a multiple of 5, cancel 5; and as 2 is now a common factor of two of the quotients, divide it into them, reserving as before; and as 15 is now a multiple of 3, cancel 3; the product of all the divisors and of the remaining quotients and original numbers, if there be any, will be the least common multiple.

3. Find the least common multiple of 4, 6, 8, and 24.

Since 24 is a multiple of each of the other numbers, it is a common multiple of all the numbers.

4. Find the least common multiple of 6, 8, and 10.
Ans. 120.

5. Find the least common multiple of 10, 12, and 14.
Ans. 420.

6. Find the least common multiple of 9, 12, 18, 27, and 36. *Ans.* 108.

7. Find the least common multiple of 7, 14, 18, 21, and 28. *Ans.* 252.

8. Find the least common multiple of 5, 15, 20, 30, and 60. *Ans.* 60.

GREATEST COMMON DIVISOR.

The ***Greatest Common Divisor*** of two or more numbers is the largest number that will exactly divide all the numbers.

PROBLEM.

Find the greatest common divisor of 4 and 10. *Ans.* 2.

2, 2. 2, 5.

The greatest common divisor of any number of terms, must be the factor, or the product of the factors, common to the terms.

EXAMPLES.

1. Find the G. C. D. of 6 and 15. *Ans.* 3.

2, 3. 3, 5.

2. Find the G. C. D. of 12 and 18. *Ans.* $2\times 3=6$.

2, 2, 3. 2, 3, 3.

3. Find the G. C. D. of 42 and 70.

Short method,

$$\begin{array}{r|rr} 2 & 42 & 70 \\ \hline 7 & 21 & 35 \\ \hline & 3 & 5 \end{array}$$

Ans. $2\times 7=14$.

CANCELLATION.

THEOREM.

The dividend contains all and exactly the same factors as the divisor and quotient.

Any composite number is the product of all its prime factors, and may be resolved into them. The product of any two integral numbers is a composite number and must contain all the factors of both numbers; and as a dividend is the product of its divisor and quotient, it must contain the same factors as its divisor and quotient.

COR. 1.—The same is true if one or both divisor and quotient be fractional; for when reduced to a common denominator, their numerators may be regarded as integral.

COR. 2.—Every factor of the divisor will cancel the same factor in the dividend.

COR. 3.—The factors which are not canceled by those of the divisor will be the factors of the quotient.

COR. 4.—Canceling a factor in the dividend divides the quotient by the same factor.

COR. 5.—Canceling a factor in the divisor multiplies the quotient by the same factor.

PROBLEMS.

1. Divide 648 by 36.

$$\frac{648}{36} = \frac{\cancel{2},\ \cancel{2},\ 2,\ 3,\ 3,\ \cancel{3},\ \cancel{3}.}{\cancel{2},\ \cancel{2},\ \cancel{3},\ \cancel{3}.} = 18,\ \textit{Ans.}$$

2. Divide 625 by 125.

$$\frac{625}{125} = \frac{\cancel{5},\ \cancel{5},\ \cancel{5},\ 5.}{\cancel{5},\ \cancel{5},\ \cancel{5}.} = 5,\ \textit{Ans.}$$

3. Divide 500 by 100.

$$\frac{5\cancel{0}\cancel{0}}{1\cancel{0}\cancel{0}} = 5,\ \textit{Ans.}$$

4. A man bought 30 yards of cloth at $5 a yard; he then exchanged it for other cloth at $3 a yard. How many yards of the latter did he get?

$\frac{\overset{10}{\cancel{30}} \times 5}{\cancel{3}} = 50$ yards. The 3 of the divisor is canceled into 30 of the dividend.

5. Sold 48 cattle at $60 each, and invested the proceeds in sheep at $5 each; how many sheep were purchased.

$$\frac{\overset{12}{\cancel{60}} \times 48}{\cancel{5}} = 576 \text{ sheep.}$$

6. A farmer sold 150 bushels of wheat at 125 cents per bushel, and invested the proceeds in oats at 25 cents per bushel; how much oats did he get?

$$\frac{\overset{5}{\cancel{125}} \times 150}{\cancel{25}} = 750 \text{ bu. oats.}$$

7. Sold 160 acres of land at $50 per acre, and bought another tract for the amount of sales, at $40 per acre; how much land was bought?

$$\frac{\$50 \times \overset{40}{\cancel{160}}}{\cancel{40}} = 200 \text{ acres.}$$

8. Divide 18 × 15 × 16 × 24 × 32 by 9, 5, 8, 12, and 16.

$$\frac{\overset{2}{\cancel{18}} \times \overset{3}{\cancel{15}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{24}} \times \overset{2}{\cancel{32}}}{\cancel{9} \times \cancel{5} \times \cancel{8} \times \cancel{12} \times \cancel{16}} = 48.$$

9. Divide 100 × 102 × 96 × 45 by 102 × 100 × 16 × 9.

$$\frac{\cancel{100} \times \cancel{102} \times \overset{6}{\cancel{96}} \times \overset{5}{\cancel{45}}}{\cancel{102} \times \cancel{100} \times \cancel{16} \times \cancel{9}} = 30.$$

FRACTIONS.

DEF. 1.—If a unit or any other number is divided into equal parts, one or more of these parts is a fraction of the whole, and all the parts constitute the whole. If a unit is divided into two equal parts, each part is called one-half, and is written $\frac{1}{2}$; and the two halves constitute the whole; thus, $\frac{2}{2} = 1$. If a unit is divided into three equal parts, each part is one-third ($\frac{1}{3}$); two of the parts, $\frac{2}{3}$; and the three thirds constitute the whole; thus, $\frac{3}{3} = 1$. If 5 is divided into two equal parts, each part is five-halves ($\frac{5}{2}$); two of the parts, $\frac{10}{2} = 5$, etc.; if 6 is divided into three equal parts, each part is $\frac{6}{3} = 2$; and two-thirds of 6 is 4, etc. Fourths, fifths, sixths, etc., are similarly constructed.

2. When a unit is divided into equal parts, any number of the parts less than the whole, expressed fractionally, is called a ***Proper Fraction;*** as, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{9}{10}$, etc. The quotient is the same in division when the dividend is less than the divisor; as, $\frac{9}{12}$, $\frac{12}{17}$, etc.; but when the divisor is less than the dividend, the quotient is called an ***Improper Fraction;*** as, $\frac{5}{4}$, $\frac{50}{6}$, $\frac{15}{7}$, etc.

3. When the division indicated by an Improper Fraction is performed, and the divisor is not contained an exact number of times in the dividend, the quotient is partly integral and partly fractional, and is termed a

Mixed Number; thus, $\frac{5}{4} = 1\frac{1}{4}$; $\frac{50}{6} = 8\frac{2}{6}$; and $\frac{75}{7} = 10\frac{5}{7}$.

Cor.—The denominator expresses the number of parts into which a unit or any other number is divided, and the numerator expresses the number of parts of a unit taken, or the number divided.

Exemplification.—If each half is divided into two equal parts, the whole number of parts is four, and the one-half has made two of those parts; hence, $\frac{1}{2} = \frac{2}{4}$; if each half is divided into three equal parts, the whole number of parts is six, and $\frac{1}{2} = \frac{3}{6}$; $\therefore \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$, etc., and $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$, etc.; hence, if both terms of a fraction are multiplied by the same number, the value of the fraction is not changed, and by this principle fractions are reduced to a Common Denominator.

PROBLEMS.

1. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator.

$$\frac{1 \times 3}{2 \times 3} = \frac{3}{6}, \quad \text{and} \quad \frac{1 \times 2}{3 \times 2} = \frac{2}{6}.$$

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to a common denominator.

$$\frac{1 \times 6}{2 \times 6} = \frac{6}{12}, \quad \frac{1 \times 4}{3 \times 4} = \frac{4}{12}, \quad \text{and} \quad \frac{1 \times 3}{4 \times 3} = \frac{3}{12}.$$

3. Reduce $\frac{4}{5}$ and $\frac{5}{7}$ to a common denominator.

$$\frac{4 \times 7}{5 \times 7} = \frac{28}{35}, \quad \text{and} \quad \frac{5 \times 5}{7 \times 5} = \frac{25}{35}.$$

Cor. 1.—The least common multiple of all the denominators is the least common denominator.

Cor. 2.—Fractions are reduced to a common denominator thus: Multiply both terms of each fraction by the quotient obtained by dividing its denominator into the least common denominator; or, when all the denom-

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Mixed Number; thus, $\frac{5}{4} = 1\frac{1}{4}$; $\frac{50}{6} = 8\frac{2}{6}$; and $\frac{75}{7} = 10\frac{5}{7}$.

Cor.—The denominator expresses the number of parts into which a unit or any other number is divided, and the numerator expresses the number of parts of a unit taken, or the number divided.

Exemplification.—If each half is divided into two equal parts, the whole number of parts is four, and the one-half has made two of those parts; hence, $\frac{1}{2} = \frac{2}{4}$; if each half is divided into three equal parts, the whole number of parts is six, and $\frac{1}{2} = \frac{3}{6}$; $\therefore$ $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$, etc., and $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$, etc.; hence, if both terms of a fraction are multiplied by the same number, the value of the fraction is not changed, and by this principle fractions are reduced to a Common Denominator.

PROBLEMS.

1. Reduce $\frac{1}{2}$ and $\frac{1}{3}$ to a common denominator.

$\frac{1\times3}{2\times3}=\frac{3}{6}$, and $\frac{1\times2}{3\times2}=\frac{2}{6}$.

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to a common denominator.

$\frac{1\times6}{2\times6}=\frac{6}{12}$, $\frac{1\times4}{3\times4}=\frac{4}{12}$, and $\frac{1\times3}{4\times3}=\frac{3}{12}$.

3. Reduce $\frac{4}{5}$ and $\frac{5}{7}$ to a common denominator.

$\frac{4\times7}{5\times7}=\frac{28}{35}$, and $\frac{5\times5}{7\times5}=\frac{25}{35}$.

Cor. 1.—The least common multiple of all the denominators is the least common denominator.

Cor. 2.—Fractions are reduced to a common denominator thus: Multiply both terms of each fraction by the quotient obtained by dividing its denominator into the least common denominator; or, when all the denominators are prime to each other, multiply both terms of each fraction by all the other denominators.

COR. 3.—An integral number is reduced to a fraction by multiplying it by the denominator of the fraction.

EXAMPLES.

1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to a common denominator.
Ans. $\frac{40}{60}$, $\frac{45}{60}$, and $\frac{48}{60}$.

2. Reduce $\frac{1}{4}$, $\frac{5}{6}$, and $\frac{3}{8}$ to a common denominator.
Ans. $\frac{6}{24}$, $\frac{20}{24}$, and $\frac{9}{24}$.

3. Reduce $\frac{5}{7}$, $\frac{7}{8}$, and $\frac{8}{9}$ to a common denominator.
Ans. $\frac{360}{504}$, $\frac{441}{504}$, and $\frac{448}{504}$.

4. Reduce $\frac{3}{5}$, $\frac{5}{7}$, $\frac{2}{9}$, and $\frac{3}{11}$ to a common denominator.
Ans. $\frac{2079}{3465}$, $\frac{2475}{3465}$, $\frac{770}{3465}$, and $\frac{945}{3465}$.

5. Reduce $\frac{1}{4}$, $\frac{6}{25}$, $\frac{7}{100}$, and $\frac{3}{1000}$ to a common denominator.
Ans. $\frac{250}{1000}$, $\frac{240}{1000}$, $\frac{70}{1000}$, and $\frac{3}{1000}$.

EXEMPLIFICATION.—Since $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}$, etc., and $\frac{1}{3}=\frac{2}{6}=\frac{3}{9}=\frac{4}{12}=\frac{5}{15}$, etc., and $\frac{2\div 2}{4\div 2}=\frac{1}{2}$, $\frac{3\div 3}{6\div 3}=\frac{1}{2}$, $\frac{4\div 4}{8\div 4}=\frac{1}{2}$, etc., and $\frac{2\div 2}{6\div 2}=\frac{1}{3}$, $\frac{3\div 3}{9\div 3}=\frac{1}{3}$, $\frac{4\div 4}{12\div 4}=\frac{1}{3}$, etc.; hence, if both terms of a fraction are divided by the same number, the value of the fraction is not changed.

COR.—When both terms of a fraction have a common factor, it may be canceled, and the fraction is thereby reduced to its lowest terms.

REM.—The common factor may be either a prime or a composite number, and it is the greatest common divisor of the terms.

EXAMPLES.

1. Reduce $\frac{9}{45}$ to its lowest terms. *Ans.* $\frac{9\div 9}{45\div 9}=\frac{1}{5}$.
2. Reduce $\frac{11}{99}$ to its lowest terms. *Ans.* $\frac{1}{9}$.
3. Reduce $\frac{13}{1703}$ to its lowest terms. *Ans.* $\frac{1}{131}$.

ADDITION AND SUBTRACTION OF FRACTIONS.

EXAMPLES.

1. Add and subtract $\frac{3}{4}$ and $\frac{1}{2}$. 4 is L. C. D.

2. Add and subtract $\frac{2}{3}$ and $\frac{1}{2}$. 6 is L. C. D.
Ans. Sum, $\frac{7}{6}$; difference, $\frac{1}{6}$.

3. Add and subtract $\frac{3}{4}$ and $\frac{3}{5}$. 20 is L. C. D.
Ans. Sum, $\frac{27}{20}$; difference, $\frac{3}{20}$.

4. Add and subtract $\frac{5}{6}$ and $\frac{4}{5}$. 30 is L. C. D.
Ans. Sum, $\frac{49}{30}$; difference, $\frac{1}{30}$.

5. Add and subtract $\frac{5}{6}$ and $\frac{5}{7}$. 42 is L. C. D.
Ans. Sum, $\frac{65}{42}$; difference, $\frac{5}{42}$.

Cor.—When the denominators have no common factor, then their product is the L. C. D., and each numerator is multiplied by all the denominators except its own

6. Add $\frac{2}{3}$ and $\frac{3}{4}$. L. C. D., 12.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}. \qquad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}.$$

$$8 + 9 = 17. \qquad \text{Sum} = \frac{17}{12} = 1\frac{5}{12}.$$

7. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$. Sum $= \frac{133}{60} = 2\frac{13}{60}$.

As no two numbers have a common factor, the L. C. D. is the product of all the denominators; and then as each denominator is multiplied by the other two denominators, so each numerator must be multiplied by the product of all the denominators except its own.

8. Add $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{7}{12}$.

$$\begin{array}{r|rrr} 4 & 5 & 8 & 12 \\ \hline & 5 & 2 & 3 \end{array} \qquad 4 \times 5 \times 2 \times 3 = 120.$$

$$\frac{120}{5} = 24. \qquad \frac{120}{8} = 15. \qquad \frac{120}{12} = 10.$$

∴ 5, 8, and 12 are the multipliers of the fractions.

9. Add and subtract $\frac{48}{275}$ and $\frac{64}{350}$.

$$\begin{array}{r|cc} 25 & 275 & 350 \\ \hline & 11 & 14 \end{array} \qquad 25 \times 11 \times 14 = 3850, \text{ L. C. D.}$$

Sum $= \frac{1376}{3850}$, and difference $= \frac{32}{3850}$.

10. Add and subtract $3\frac{2}{3}$ and $2\frac{1}{4}$.

$$3\frac{2}{3} = 3\frac{8}{12}$$
$$2\frac{1}{4} = 2\frac{3}{12}$$

Difference $= 1\frac{5}{12}$. Sum $= 5\frac{11}{12}$.

$$\text{Or,}\quad 3\frac{2}{3} = \frac{11 \times 4}{3 \times 4} = \frac{44}{12}$$
$$2\frac{1}{4} = \frac{9 \times 3}{4 \times 3} = \frac{27}{12}$$

Sum $= \frac{71}{12} = 5\frac{11}{12}$. Difference $= \frac{17}{12} = 1\frac{5}{12}$.

Rem.—Mixed numbers may be united by either of the above methods; the latter is generally preferred.

11. Cast up the following account:

			cents.
Bought	3 pairs hose,	at 37½ cts. =	112½
	5 pocket handkerchiefs,	at 62½ cts. =	312½
	3 pocket knives,	at 31¼ cts. =	93¾
	7 lbs. sugar,	at 12½ cts. =	87½
	9 inkstands,	at 6¼ cts. =	56¼
	5 qrs. paper,	at 15¼ cts. =	76¼
			738¾
	Paid on account,		549¼
	Balance due,		189½ cts.

The numerators of the 4ths show the value, but those of the halves must be doubled, as 4 is the common denominator.

MULTIPLICATION OF FRACTIONS.

THEOREM.

The product of two proper fractions is less than either fraction.

For, if a number is multiplied by one, the product is the same as the number multiplied. If the multiplier is greater than one, the product is greater than the number; and if the multiplier is less than one, the product is less than the number.

In the multiplication of two proper fractions, each factor is less than one; hence the product is less than either fraction.

PROBLEMS.

1. Multiply $\frac{2}{3}$ by 1. *Ans.* $\frac{2}{3} \times 1 = \frac{2}{3}$.
2. Multiply $\frac{2}{3}$ by 2. *Ans.* $\frac{2}{3} \times 2 = \frac{4}{3}$.
3. Multiply $\frac{2}{3}$ by $\frac{1}{2}$. *Ans.* $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

It is evident that $\frac{2}{3}$ multiplied by $1 = \frac{2}{3}$, that $\frac{2}{3} \times 2$ is $\frac{4}{3}$, and $\frac{2}{3} \times \frac{1}{2}$ or $\frac{1}{2}$ time $\frac{2}{3}$ is $\frac{1}{3}$; and, as alternating the factors does not change the product, therefore, $1 \times \frac{2}{3} = \frac{2}{3}$, $2 \times \frac{2}{3} = \frac{4}{3}$, and $\frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$.

4. Multiply $\frac{7}{5}$ by $\frac{8}{7}$. *Ans.* $\frac{\cancel{7}}{5} \times \frac{8}{\cancel{7}} = \frac{8}{5}$.

Cor. 1.—In multiplying by a fraction the numerator is a multiplier and the denominator a divisor.

Cor. 2.—In the multiplication of fractions, the product of all the numerators is the numerator of the product; and the product of all the denominators is the denominator of the product.

Cor. 3.—The product of two improper fractions is greater than either fraction.

EXAMPLES.

1. Multiply $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$, $\frac{10}{11}$, $\frac{11}{12}$.

$$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{6}}{\cancel{7}} \times \frac{\cancel{7}}{\cancel{8}} \times \frac{\cancel{8}}{\cancel{9}} \times \frac{\cancel{9}}{\cancel{10}} \times \frac{\cancel{10}}{\cancel{11}} \times \frac{\cancel{11}}{12} = \frac{1}{12}.$$

By analysis, $\frac{1}{2}$ of $\frac{2}{3} = \frac{1}{3}$, $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{4}$, $\frac{1}{4}$ of $\frac{4}{5} = \frac{1}{5}$, $\frac{1}{5}$ of $\frac{5}{6} = \frac{1}{6}$, $\frac{1}{6}$ of $\frac{6}{7} = \frac{1}{7}$, $\frac{1}{7}$ of $\frac{7}{8} = \frac{1}{8}$, $\frac{1}{8}$ of $\frac{8}{9} = \frac{1}{9}$, $\frac{1}{9}$ of $\frac{9}{10} = \frac{1}{10}$, $\frac{1}{10}$ of $\frac{10}{11} = \frac{1}{11}$, $\frac{1}{11}$ of $\frac{11}{12} = \frac{1}{12}$.

2. Multiply $\frac{7}{9}$, $\frac{10}{11}$, and $\frac{3}{5}$; thus,

$$\frac{7}{\underset{3}{\cancel{9}}} \times \frac{\overset{2}{\cancel{10}}}{11} \times \frac{\cancel{3}}{\cancel{5}} = \frac{14}{33}, \text{ product.}$$

3. Multiply $\frac{9}{16}$ and $\frac{3}{4}$.

$$\frac{9}{16} \times \frac{3}{4} = \frac{27}{64}, \text{ product.}$$

4. Multiply $35\frac{3}{4}$ by 9.

$$\begin{array}{rcrr} \frac{3}{4} \times 9 & = \frac{27}{4} & = & 6\frac{3}{4} \\ 35 \times 9 & & = & \underline{315} \\ & & & 321\frac{3}{4} = \text{Product.} \end{array}$$

Axiom 7.—If any number be both multiplied and divided by the same number, the value of the original number is not changed.

Cor. 1.—If the multiplier is greater than the divisor, the product is greater than the number multiplied, but if the multiplier is less than the divisor, then the product is less than the multiplicand.

Cor. 2.—Multiplying the numerator or dividing the denominator by any number, multiplies the fraction by the same number.

5. Multiply 37 by $8\frac{1}{2}$.

$$\begin{array}{rr} & 37 \\ & \underline{8\frac{1}{2}} \\ & 18\frac{1}{2} \\ & \underline{296} \\ \text{Product} = & 314\frac{1}{2} \end{array}$$

6. Multiply $37\frac{3}{4}$ by $11\frac{2}{3}$.

$$37\frac{3}{4} \times 11\frac{2}{3} = \frac{151}{4} \times \frac{35}{3} = \frac{5285}{12} = 440\frac{5}{12} = \text{Product.}$$

REM.—When only one of the factors is a mixed number, they may be solved as the 4th and 5th examples; but when both factors are mixed numbers, it is better to reduce them to improper fractions as in 6th example.

7. Multiply $14\frac{2}{3}$ by 4.
8. Multiply $17\frac{1}{6}$ by 6.
9. Multiply $18\frac{1}{4}$ by 9.
10. Multiply $24\frac{3}{4}$ by 8.
11. Multiply 1456 by $15\frac{1}{4}$.
12. Multiply 2375 by $27\frac{1}{3}$.
13. Multiply 3147 by $35\frac{3}{4}$.
14. Multiply $4156\frac{3}{4}$ by $2124\frac{2}{3}$.
15. Multiply $127\frac{5}{11}$ by $13\frac{3}{7}$.
16. Multiply $15\frac{4}{9}$ by $9\frac{2}{7}$.
17. Multiply $29\frac{3}{37}$ by $13\frac{2}{21}$.
18. Multiply $104\frac{2}{3}$ by $20\frac{4}{9}$.

DIVISION OF FRACTIONS.

PROBLEMS.

1. Divide 10 by 5. *Ans.* $10 \div 5 = \frac{10}{5} = 2$.
2. Divide 6 by 3. *Ans.* $\frac{6}{3} = 2$.
3. Divide 3 by 3. *Ans.* $\frac{3}{3} = 1$.
4. Divide 1 by 2. *Ans.* $\frac{1}{2}$.
5. Divide 1 by 3. *Ans.* $\frac{1}{3}$.
6. Divide 2 by 3. *Ans.* $\frac{2}{3}$.
7. Divide 3 by 4. *Ans.* $\frac{3}{4}$.
8. Divide 5 by 4. *Ans.* $\frac{5}{4} = 1\frac{1}{4}$.
9. Divide $\frac{1}{2}$ by 2, or divide $\frac{1}{2}$ into two equal parts. *Ans.* $\frac{1}{2\times 2} = \frac{1}{4}$.

10. Divide $\frac{2}{3}$ by 2, or divide $\frac{2}{3}$ into two equal parts.
Ans. $\frac{2\div 2}{3} = \frac{1}{3}$.

11. Divide $\frac{1}{3}$ by 2, or divide $\frac{1}{3}$ into two equal parts.
Ans. $\frac{1}{3\times 2} = \frac{1}{6}$.

12. Divide $\frac{2}{3}$ by $\frac{1}{3}$, or how often is $\frac{1}{3}$ contained in $\frac{2}{3}$.
Ans. Evidently twice.

13. Divide $\frac{1}{2}$ by $\frac{1}{4}$, or how often is $\frac{1}{4}$ contained in $\frac{1}{2} = \frac{2}{4}$.
Ans. Evidently twice.

14. Divide $\frac{1}{2}$ by $\frac{1}{3}$, or how often is $\frac{1}{3}$ contained in $\frac{1}{2}$.
Ans. $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$; $\frac{3}{6} \div \frac{2}{6} = \frac{3}{2} = 1\frac{1}{2}$.

15. Divide $\frac{1}{5}$ by $\frac{1}{4}$, or how often is $\frac{1}{4}$ contained in $\frac{1}{5}$.
Ans. $\frac{1}{5} = \frac{4}{20}$ and $\frac{1}{4} = \frac{5}{20}$; $\frac{4}{20} \div \frac{5}{20} = \frac{4}{5}$.

COR. 1.—The numerator of a fraction is a dividend, the denominator a divisor, and the fraction itself the quotient.

COR. 2.—To divide a fraction by a fraction, reduce both to a common denominator and then divide the numerator of the dividend by the numerator of the divisor.

COR. 3.—Dividing the numerator or multiplying the denominator by any number, divides the fraction by the same number.

THEOREM.

To divide any number by a fraction, invert the fractional divisor and make it a multiplier.

1st. When the dividend is integral.

If the dividend is multiplied by the denominator of the divisor, the product will be the numerator of the quotient, and the numerator of the divisor will be the denominator.

2d. When the dividend is also a fraction.

If the numerator of the dividend is multiplied by the denominator of the divisor, the product will be the numerator of the quotient, and if the numerator of the divisor is multiplied by the denominator of the dividend, the product will be the denominator of the quotient.

PROBLEMS.

1. Divide 15 by $\frac{3}{4}$.

$$15 \times 4 = \tfrac{60}{4}, \quad \text{and} \quad \tfrac{60}{4} \div \tfrac{3}{4} = \tfrac{60}{3} = 20.$$

$$15 \times \tfrac{4}{3} = 20.$$

2. Divide $\frac{15}{3}$ by $\frac{3}{4}$.

$$\tfrac{15}{3} \times \tfrac{4}{4} = \tfrac{60}{12}, \quad \text{and} \quad \tfrac{3}{4} \times \tfrac{3}{3} = \tfrac{9}{12}, \quad \text{and} \quad \tfrac{60}{9} = \tfrac{20}{3} = 6\tfrac{2}{3}.$$

$$\tfrac{15}{3} \times \tfrac{4}{3} = \tfrac{60}{9} = \tfrac{20}{3} = 6\tfrac{2}{3}.$$

Any number is divided by a fraction by inverting the fractional divisor and making it a multiplier.

EXAMPLES

1. Divide 36 by $\frac{4}{5}$.
2. Divide 12 by $\frac{2}{3}$.
3. Divide 16 by $\frac{4}{7}$.
4. Divide $\frac{3}{4}$ by $\frac{5}{6}$.
5. Divide $\frac{1}{2}$ by $\frac{2}{3}$.
6. Divide $\frac{2}{3}$ by $\frac{1}{2}$.
7. Divide $112\frac{1}{2}$ by $21\frac{2}{3}$.
8. Divide $14\frac{2}{3}$ by $9\frac{1}{6}$.
9. Divide $33\frac{1}{3}$ by $6\frac{1}{3}$.
10. Divide 15 by $\frac{7}{15}$.
11. Divide $\frac{15}{4}$ by $\frac{7}{15}$.
12. Divide $\frac{15}{19}$ by $\frac{2}{7}$.
13. Divide $\frac{13}{15}$ by $\frac{9}{11}$.
14. Divide $208\frac{3}{4}$ by $27\frac{5}{9}$.
15. Divide $109\frac{1}{8}$ by $29\frac{2}{7}$.

REM.—Mixed numbers should be reduced to improper fractions.

16. Divide $5436\frac{3}{7}$ by 3. *Ans.* $1812\frac{1}{7}$.

17. Divide $6,478\frac{3}{5}$ by 9. *Ans.* $719\frac{38}{45}$.

$$9\,)\underline{6478\frac{3}{5}}$$
$$719\text{, remainder } 7\frac{3}{5}=\frac{38}{5} \text{ and } \frac{38}{5}\div 9=\frac{38}{45}.$$

18. Divide $250\frac{4}{9}$ by $12\frac{2}{3}$. *Ans.* $19\frac{44}{57}$.

19. Divide 450 by $\frac{3}{4}$. *Ans.* 600.

20. Divide $\frac{4}{5}$ by 20. *Ans.* $\frac{1}{25}$.

21. Divide $\frac{4}{5}\times\frac{5}{6}\times\frac{6}{7}\times\frac{7}{8}$ by $\frac{8}{9}$, $\frac{9}{10}$, $\frac{10}{11}$, $\frac{11}{12}$. *Ans.* $\frac{3}{4}$.

22. Divide $\frac{3}{4}$ by $\frac{1}{4}$. *Ans.* 3.

23. Divide $\frac{5}{7}$ by $\frac{1}{7}$. *Ans.* 5.

24. Divide 1 by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. *Ans.* 2, 3, 4, 5.

25. Divide 1 by $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. *Ans.* $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{4}$.

26. Divide 2 by $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$. *Ans.* $\frac{6}{2}$, $\frac{8}{3}$, $\frac{10}{4}$.

Cor. 1. Any fraction having unity for its numerator is contained in 1 as many times as there are units in its denominator; thus, $\frac{1}{2}$ is contained in 1 twice, $\frac{1}{3}$ three times, &c. If the fraction has 2 for its numerator, it is contained one-half as many times; if the numerator is 3, one-third as many times, &c.

Cor. 2. If the dividend is increased, the quotient is increased just as many times; if the divisor is increased, the quotient is diminished just as many times; the converse is also true in both cases.

Cor. 3. To divide any number, either integral or fractional, by a fraction, invert the divisor and multiply the dividend by it.

COMPLEX FRACTIONS.

When one or both terms of a fraction are either fractions or mixed numbers, it is called a ***Complex Fraction;*** thus,

$$\frac{3\frac{1}{2}}{5},\quad \frac{\frac{1}{2}}{\frac{2}{3}},\quad \frac{3}{\frac{2}{5}},\quad \frac{37\frac{1}{2}}{62\frac{1}{2}},\quad \frac{100\frac{1}{2}}{100},$$ etc., are complex fractions.

Rem.—When we consider that the numerator of a fraction represents a dividend, and the denominator a divisor, a complex fraction is readily reduced to a simple fraction.

1. Reduce $\frac{3\frac{1}{2}}{5}$ to a simple fraction.

$$\frac{3\frac{1}{2}}{5} = \frac{\frac{7}{2}}{5} = \frac{7}{2} \times \frac{1}{5} = \frac{7}{10}.$$

Rem.—As the denominator is a divisor, it must be inverted.

2. Reduce $\frac{\frac{1}{2}}{\frac{2}{3}}$ to a simple fraction.

$$\frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}.$$

3. Reduce $\frac{3}{\frac{2}{5}}$ to a common fraction.

$$\frac{3}{\frac{2}{5}} = 3 \times \frac{5}{2} = \frac{15}{2} = 7\frac{1}{2}.$$

3. Reduce $\frac{37\frac{1}{2}}{62\frac{1}{2}}$ to a common fraction.

$$\frac{37\frac{1}{2}}{62\frac{1}{2}} = \frac{\frac{75}{2}}{\frac{125}{2}} = \frac{\overset{3}{\cancel{75}}}{\cancel{2}} \times \frac{\cancel{2}}{\underset{5}{\cancel{125}}} = \frac{3}{5}.$$

5. Multiply $\frac{100\frac{1}{2}}{100\frac{1}{4}} \times \frac{40\frac{1}{3}}{60\frac{1}{2}} = \frac{\frac{201}{2}}{\frac{401}{4}} \times \frac{\frac{121}{3}}{\frac{121}{2}} = \frac{\overset{67}{\cancel{201}}}{\cancel{2}} \times \frac{\cancel{121}}{\cancel{3}} \times$
$\frac{4}{401} \times \frac{\cancel{2}}{\cancel{121}} = \frac{268}{401}.$

6. Divide $\frac{\frac{1}{2}}{\frac{2}{3}}$ by $\frac{\frac{3}{4}}{\frac{4}{5}} = \frac{\frac{1}{2}}{\frac{2}{3}} \times \frac{\frac{4}{5}}{\frac{3}{4}} = \frac{1}{2} \times \frac{\overset{2}{\cancel{4}}}{5} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{4}{\cancel{3}} = \frac{4}{5}.$
$\frac{1}{2} \times \frac{4}{5} \times \frac{3}{2} \times \frac{4}{3} = \frac{4}{5}.$

In changing the division to multiplication, the whole divisor must be inverted; that is, $\frac{4}{5}$ becomes the numerator and $\frac{3}{4}$ becomes the denominator, with the sign of multiplication; then again the two denominators must be inverted; then they are all in the form of multiplication.

REVIEW.

REDUCTION.

PROBLEM.—Reduce $\frac{2}{3}$, $\frac{3}{5}$, and $\frac{5}{7}$ to a common denominator.

Since all the denominators are prime to each other, the common denominator is their product; thus, $3 \times 5 \times 7 = 105$, the denominator;

And $2 \times 5 \times 7 = 70$; $3 \times 3 \times 7 = 63$; $5 \times 3 \times 5 = 75$, the numerators;

Or $\frac{2}{3} \times \frac{5}{5} \times \frac{7}{7} = \frac{70}{105}$; $\frac{3}{5} \times \frac{3}{3} \times \frac{7}{7} = \frac{63}{105}$; $\frac{5}{7} \times \frac{3}{3} \times \frac{5}{5} = \frac{75}{105}$.

COR. Each numerator is multiplied by all the denominators but its own.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{5}{6}$ to a common denominator.
Ans. 6 is the L. C. D.; $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$.

2. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ to a common denominator.
Ans. L. C. D., 60; $\frac{20}{60}$, $\frac{45}{60}$, $\frac{48}{60}$.

3. Reduce $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{7}$ to a common denominator.
Ans. $\frac{105}{140}$, $\frac{112}{140}$, $\frac{100}{140}$.

4. Reduce $\frac{4}{9}$, $\frac{5}{11}$, and $\frac{6}{13}$ to a common denominator.
Ans. $\frac{572}{1287}$, $\frac{585}{1287}$, $\frac{594}{1287}$.

5. Reduce $\frac{7}{12}$, $\frac{11}{15}$, $\frac{13}{20}$, and $\frac{17}{24}$, to a common denominator.
Ans. $\frac{70}{120}$, $\frac{88}{120}$, $\frac{78}{120}$, $\frac{85}{120}$.

6. Reduce $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, and $\frac{13}{24}$ to a common denominator.
Ans. $\frac{18}{24}$, $\frac{20}{24}$, $\frac{21}{24}$, $\frac{22}{24}$, $\frac{13}{24}$.

7. Reduce $\frac{5}{17}$, $\frac{7}{221}$ to a common denominator.
Ans. $\frac{65}{221}$, $\frac{7}{221}$.

ADDITION.

PROBLEMS

1. What is the sum of $\frac{1}{2}$ and $\frac{1}{3}$?	*Ans.* $\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$.
2. What is the sum of $\frac{1}{3}$ and $\frac{1}{4}$?	*Ans.* $\frac{7}{12}$.
3. What is the sum of $\frac{1}{5}$ and $\frac{1}{6}$?	*Ans.* $\frac{11}{30}$.
4. What is the sum of $\frac{1}{7}$ and $\frac{1}{8}$?	*Ans.* $\frac{15}{56}$.

COR. The sum of any two fractions whose numerators are unity, will have for its numerator the sum of its denominators, and for its denominator their product.

5. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{5}$?	*Ans.* $\frac{31}{30}$.
6. What is the sum of $\frac{1}{6}$ $\frac{1}{7}$, and $\frac{1}{11}$?	*Ans.* $\frac{185}{462}$.

COR. The sum of any three fractions whose numerators are unity will have for its numerator the sum of the products of the denominators, taken two at a time, and for its denominator the product of all the denominators.

EXAMPLES.

1. Add $\frac{3}{7}$, $\frac{5}{9}$, $\frac{7}{11}$, and $\frac{9}{13}$.	*Ans.* $2\frac{2818}{9009}$.
2. Add $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, and $\frac{11}{12}$.	*Ans.* $4\frac{11}{40}$.
3. Add $\frac{11}{12}$, $\frac{12}{13}$.	*Ans.* $1\frac{131}{156}$.
4. Add $\frac{12}{13}$, $\frac{13}{12}$.	*Ans.* $2\frac{1}{156}$.
5. Add $\frac{2}{9}$, $\frac{5}{18}$, $\frac{7}{27}$, and $\frac{11}{36}$.	*Ans.* $1\frac{7}{108}$.
6. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$, $\frac{9}{32}$.	*Ans.* $2\frac{3}{32}$.
7. Add $\frac{4}{5}$, $\frac{7}{15}$, $\frac{3}{25}$, $\frac{4}{75}$.	*Ans.* $1\frac{11}{25}$.
8. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{24}=$	*Ans.* $\frac{36}{24}=1\frac{1}{2}$.
9. Add $2\frac{1}{2}$, $3\frac{2}{3}$, $4\frac{1}{4}$, and $5\frac{3}{5}$.	*Ans.* $16\frac{1}{60}$.
10. Add $12\frac{2}{3}$, $15\frac{3}{4}$, and $25\frac{4}{5}$.	*Ans.* $54\frac{13}{60}$.
11. Add $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.	*Ans.* $2\frac{43}{60}$.
12. Add $\frac{5}{12}$, $\frac{7}{16}$, $\frac{11}{24}$, and $\frac{13}{32}$.	*Ans.* $1\frac{23}{32}$.
13. Add $\frac{1}{7}$, $\frac{2}{9}$, and $\frac{3}{11}$.	*Ans.* $\frac{442}{693}$.
14. Add $\frac{21}{541}$ and $\frac{37}{253}$.	*Ans.* $\frac{25330}{136873}$.

SUBTRACTION.

PROBLEMS.

1. From $\frac{2}{3}$ take $\frac{1}{2}$. *Ans.* $\frac{1}{6}$. 4. From $\frac{5}{6}$ take $\frac{4}{5}$. *Ans.* $\frac{1}{30}$.
2. From $\frac{3}{4}$ take $\frac{2}{3}$. *Ans.* $\frac{1}{12}$. 5. From $\frac{6}{7}$ take $\frac{5}{6}$. *Ans.* $\frac{1}{42}$.
3. From $\frac{4}{5}$ take $\frac{3}{4}$. *Ans.* $\frac{1}{20}$. 6. From $\frac{7}{8}$ take $\frac{6}{7}$. *Ans.* $\frac{1}{56}$.

Cor. Two fractions whose numerators differ by unity, and each denominator is greater by unity than its numerator, the fraction having the larger terms is the greater, and their difference has unity for its numerator, and the product of the two denominators for its denominator.

EXAMPLES.

1. From $\frac{3}{4}$ take $\frac{1}{2}$. *Ans.* $\frac{1}{4}$.
2. From $1\frac{1}{2}$ take $\frac{3}{4}$. *Ans.* $\frac{3}{4}$.
3. From $5\frac{2}{3}$ take $\frac{7}{8}$. *Ans.* $4\frac{19}{24}$.
4. From $5\frac{1}{2}$ take $2\frac{3}{4}$. *Ans.* $2\frac{3}{4}$.
5. From $5\frac{5}{6}$ take $3\frac{4}{5}$. *Ans.* $2\frac{1}{30}$.
6. From $5\frac{4}{5}$ take $4\frac{5}{6}$. *Ans.* $\frac{29}{30}$.
7. From $6\frac{7}{8}$ take $4\frac{3}{4}$. *Ans.* $2\frac{1}{8}$.
8. From $12\frac{4}{5}$ take $5\frac{3}{4}$. *Ans.* $7\frac{1}{20}$.
9. From $15\frac{8}{9}$ take $12\frac{7}{8}$. *Ans.* $3\frac{1}{72}$.
10. From $37\frac{3}{4}$ take $21\frac{2}{3}$. *Ans.* $16\frac{1}{12}$.
11. From $45\frac{1}{3}$ take $34\frac{1}{4}$. *Ans.* $11\frac{1}{12}$.
12. From $27\frac{4}{5}$ take $15\frac{5}{6}$. *Ans.* $11\frac{29}{30}$.
13. From a piece of cloth containing $16\frac{2}{3}$ yards, a merchant sold $7\frac{3}{4}$ yds.; how much was left? *Ans.* $8\frac{11}{12}$ yd.
14. A merchant has 2 pieces of muslin, the one has $41\frac{2}{3}$ yards, and the other $43\frac{3}{4}$ yd.; he sells $61\frac{1}{2}$ yd.; how much has he left? *Ans.* $23\frac{11}{12}$.
15. A man earned \$$36\frac{2}{3}$, and spent \$$15\frac{2}{5}$; how much had he left? *Ans.* \$$21\frac{4}{15}$.

16. James had \$$45\frac{3}{4}$, and lost \$$16\frac{4}{5}$; how much had he left? *Ans.* \$$28\frac{19}{20}$.

17. Subtract $9\frac{4}{11}$ from $10\frac{3}{14}$. *Ans.* $\frac{131}{154}$.

18. Subtract $15\frac{6}{7}$ from $25\frac{7}{8}$. *Ans.* $10\frac{1}{56}$.

19. Subtract $36\frac{10}{11}$ from $45\frac{11}{12}$. *Ans.* $9\frac{1}{132}$.

20. Subtract $56\frac{4}{5}$ from $97\frac{3}{4}$. *Ans.* $40\frac{19}{20}$.

21. Subtract $21\frac{8}{9}$ from $35\frac{7}{8}$. *Ans.* $13\frac{71}{72}$.

22. Subtract $105\frac{6}{11}$ from $205\frac{5}{11}$. *Ans.* $99\frac{10}{11}$.

ORAL EXERCISES.

What is $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{1}{4}$ of $\frac{4}{5}$? $\frac{1}{5}$ of $\frac{5}{6}$? $\frac{1}{6}$ of $\frac{6}{7}$? $\frac{1}{7}$ of $\frac{7}{8}$? $\frac{1}{8}$ of $\frac{8}{9}$? $\frac{1}{9}$ of $\frac{9}{10}$?

One-third of 6 is $\frac{1}{2}$ of what? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

One-third of 9 is $\frac{1}{2}$ of what? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

Two-thirds of 9 is $\frac{1}{2}$ of what? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

One-third of 12 is $\frac{1}{2}$ of what? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

Two-thirds of 12 is $\frac{1}{2}$ of what? $\frac{1}{4}$? $\frac{1}{5}$? $\frac{1}{6}$? $\frac{1}{7}$? $\frac{1}{8}$? $\frac{1}{9}$? $\frac{1}{10}$? $\frac{1}{11}$? $\frac{1}{12}$?

What part of 5 is 1? 2? 3? 4? *Ans.* $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$.

What part of $5\frac{1}{2}$ is $\frac{1}{2}$? $1\frac{1}{2}$? $2\frac{1}{2}$? $3\frac{1}{2}$?
Ans. $\frac{1}{11}$, $\frac{3}{11}$, $\frac{5}{11}$, $\frac{7}{11}$.

What part of $5\frac{1}{2}$ is $\frac{1}{4}$? $1\frac{1}{4}$? $2\frac{1}{4}$? $3\frac{1}{4}$?
Ans. $\frac{1}{22}$, $\frac{5}{22}$, $\frac{9}{22}$, $\frac{13}{22}$.

What part of $5\frac{1}{2}$ is $\frac{1}{3}$? $\frac{2}{3}$? $1\frac{1}{3}$? $2\frac{1}{3}$?
Ans. $\frac{2}{33}$, $\frac{4}{33}$, $\frac{8}{33}$, $\frac{14}{33}$.

What part of 100 is $6\frac{1}{4}$? $12\frac{1}{2}$? 25? $37\frac{1}{2}$?
Ans. $\frac{1}{16}$, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$.

What part of 24 hours is 5 h.? 6 h.? $8\frac{1}{2}$ h.? $9\frac{1}{3}$ h.? *Ans.* $\frac{5}{24}$, $\frac{1}{4}$, $\frac{17}{48}$, $\frac{7}{18}$.

What part of 1 is $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? *Ans.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

What part of $\frac{3}{4}$ is $\frac{1}{2}$? $\frac{1}{3}$? $\frac{1}{4}$? $\frac{1}{5}$? *Ans.* $\frac{2}{3}$, $\frac{4}{9}$, $\frac{1}{3}$, $\frac{4}{15}$.

PROBLEMS.

1. Multiply 1 by 2. *Ans.* $1\times2=2$.
2. Multiply 1 by 1. *Ans.* $1\times1=1$.
3. Multiply 1 by $\frac{1}{2}$. *Ans.* $1\times\frac{1}{2}=\frac{1}{2}$.
4. Multiply 1 by $\frac{1}{4}$. *Ans.* $1\times\frac{1}{4}=\frac{1}{4}$.
5. Multiply 1 by $\frac{3}{4}$. *Ans.* $1\times\frac{3}{4}=\frac{3}{4}$.
6. Multiply $\frac{1}{2}$ by $\frac{3}{4}$. *Ans.* $\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$.
7. Divide 2 by 2. *Ans.* $2\div2=1$.
8. Divide 2 by 1. *Ans.* $2\div1=2$.
9. Divide 2 by $\frac{1}{2}$. *Ans.* $2\times\frac{2}{1}=4$.
10. Divide 2 by $\frac{1}{4}$. *Ans.* $2\times\frac{4}{1}=8$.
11. Divide 2 by $\frac{3}{4}$. *Ans.* $2\times\frac{4}{3}=\frac{8}{3}=2\frac{2}{3}$.
12. Divide $\frac{1}{2}$ by $\frac{3}{4}$. *Ans* $\frac{1}{2}\times\frac{4}{3}=\frac{4}{6}=\frac{2}{3}$.
13. If $\frac{1}{2}$ cord of wood cost $2, what will 1 cord cost?
Ans. $2\times\frac{2}{1}=\$4$.
14. If $\frac{1}{4}$ cord of wood cost $1, what will 1 cord cost?
Ans. $1\times\frac{4}{1}=\$4$.
15. If $\frac{3}{4}$ cord of wood cost $3, what will 1 cord cost?
Ans. $3\times\frac{4}{3}=\$4$.
16. If $\frac{5}{6}$ ton of coal cost $5, what will 1 ton cost?
Ans. $5\times\frac{6}{5}=\$6$.
17. If $\frac{7}{8}$ ton of coal cost $7, what will 1 ton cost?
Ans. $7\times\frac{8}{7}=\$8$.
18. If $\frac{1}{2}$ cord of wood cost $2, what will 3 cords cost?
Ans. $2\times\frac{2}{1}\times3=\$12$.
19. If $\frac{1}{4}$ cord of wood cost $1, what will 5 cords cost?
Ans. $4\times\frac{4}{1}\times\frac{5}{1}=\$20$.

20. If $\frac{3}{4}$ cord of wood cost $3, what will 6 cords cost?

Ans. $3 \times \frac{4}{3} \times 6 = \$24$.

21. If 3 cords wood cost $15, what will 8 cords cost?

Ans. $15 \div 3 \times 8 = \$40$.

Cor. 1. When a given number of articles, each of the same value, cost a certain sum of money, in order to find the cost of one article, divide the certain sum by the given number, whether integral or fractional.

Cor. 2. Multiply the cost of one article by any required number of articles, and the product will be the cost of that number.

EXAMPLES.

1. If $\frac{1}{7}$ of a yard of cloth cost 40 cents, what will a yard cost? what will $\frac{2}{3}$ of a yard cost? $\frac{3}{4}$? $\frac{5}{14}$? $\frac{11}{28}$?

Ans. 280 cents, 186$\frac{2}{3}$ cts., 210 cts., 100 cts., 110 cts.

2. At $9 a barrel, what will $\frac{2}{3}$ of a barrel of flour cost?

Ans. $6.

3. At $8 a ton, what will $\frac{3}{4}$ of a ton of coal cost?

Ans. $6.

4. If $\frac{2}{5}$ of a yard of linen cost 18 cts., what will 1 yard cost? *Ans.* 45 cents.

5. If $\frac{3}{7}$ of a yard of linen cost 21 cts., what will 1 yard cost? *Ans.* 49 cents.

6. If $\frac{9}{10}$ of a yard of linen cost 36 cts., what will $\frac{1}{4}$ yard cost? *Ans.* 10 cents.

7. If $\frac{3}{4}$ of a yard of linen cost 15 cts., what will 1 yard cost? *Ans.* 20 cents.

8. If $\frac{2}{3}$ of a yard of linen cost 24 cts., what will $\frac{5}{6}$ yard cost? *Ans.* 30 cents.

9. If $\frac{3}{4}$ of a yard of cloth cost $\$\frac{7}{8}$, what will 1 yard cost? 2 yd.? 3 yd.? 5 yd.? *Ans.* $\$1\frac{1}{6}$, $\$2\frac{1}{3}$, $\$3\frac{1}{2}$, $\$5\frac{5}{6}$.

10. If $\frac{7}{8}$ of a ton of hay cost \$7, what will 1 ton cost? $2\frac{3}{4}$ tons? $3\frac{5}{8}$? $7\frac{1}{2}$? $9\frac{1}{4}$? *Ans.* \$8, \$22, \$29, \$60, \$74.

11. What is the value of $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \div \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9}$. *Ans.* $\frac{9}{25}$.

12. What is the number whose factors are $\frac{11}{12}$ and $\frac{9}{11}$? *Ans.* $\frac{3}{4}$.

13. The divisor is $\frac{5}{7}$ and the quotient $\frac{3}{5}$; what is the dividend? *Ans.* $\frac{3}{7}$.

14. In what time will a man make the round of the earth—24,894 miles—by traveling $33\frac{1}{3}$ miles a day? *Ans.* $746\frac{41}{50}$ days.

15. In one mile there are 1,760 yards; how many yards in $5\frac{3}{4}$ miles? $7\frac{1}{8}$ miles? $9\frac{3}{5}$ miles? $12\frac{5}{6}$ miles?
Ans. 10,120 yds., 12,540 yds., 16,896 yds., $22,586\frac{2}{3}$ yds.

16. In one square mile there are 640 acres; how many acres in $3\frac{3}{5}$ sq. m.? $5\frac{3}{4}$ sq. m.? $9\frac{3}{8}$ sq. m.? $12\frac{1}{6}$ sq. m.? *Ans.* 2,304 A., 3,680 A., 6,000 A., $7,786\frac{2}{3}$ A.

#17. If $\frac{3}{4}$ of a ton of coal cost \$9, what will $15\frac{1}{3}$ tons cost? *Ans.* \$184.

× 18. If $\frac{2}{7}$ of a ton of coal cost \$4, what will $31\frac{1}{2}$ tons cost? *Ans.* \$441.

19. Divide $\frac{27}{35}$ by $\frac{9}{7}$, and multiply the quotient by $24\frac{2}{5}$. *Ans.* $14\frac{16}{25}$.

20. Divide $\frac{39}{78}$ by $\frac{13}{12}$, and then multiply by $\frac{78}{108}$. *Ans.* $\frac{1}{3}$.

21. Reduce $\dfrac{\frac{3}{4}}{\frac{4}{5}}$ to a simple fraction. *Ans.* $\frac{15}{16}$.

22. Reduce $\dfrac{\frac{3}{4}+\frac{4}{5}}{\frac{5}{6}+\frac{6}{7}}$ to a simple fraction. *Ans.* $\frac{651}{710}$.

23. Reduce $\dfrac{\frac{4}{5}-\frac{3}{4}}{\frac{6}{7}-\frac{5}{6}}$ to a simple fraction. *Ans.* $2\frac{1}{10}$.

24. Reduce $\frac{\frac{4}{5}}{\frac{6}{7}} \times \frac{\frac{5}{6}}{\frac{7}{8}}$ to a simple fraction. *Ans.* $\frac{8}{9}$.

25. Reduce $\frac{\frac{8}{9}}{\frac{10}{11}} \div \frac{\frac{11}{12}}{\frac{9}{10}}$ to a simple fraction. *Ans.* $\frac{24}{25}$.

PRACTICAL EXAMPLES.

1. A man owns $\frac{3}{16}$ of a gold mine, and sells $\frac{2}{3}$ of his share for \$50,000. What is the whole mine worth at that rate? *Ans.* \$400,000.

2. The distance from Baltimore to Philadelphia is 97 miles; A starts from Baltimore to Philadelphia at the same time that B starts from Philadelphia to Baltimore; A travels $6\frac{1}{2}$ miles per hour, and B $7\frac{1}{4}$ miles. In what time will they meet, and how far will each have traveled?

3. What number multiplied by $\frac{7}{9}$ will give a product of $\frac{9}{11}$? *Ans.* $\frac{81}{77}$.

4. What number divided by $\frac{6}{7}$ will give a quotient of $\frac{7}{9}$? *Ans.* $\frac{2}{3}$.

5. If a man travel $237\frac{4}{11}$ miles in $21\frac{3}{7}$ hours, how far does he travel each hour? *Ans.* $11\frac{127}{1650}$.

6. If a traveler perform a journey in $17\frac{2}{5}$ hours, by traveling $6\frac{3}{7}$ miles in an hour, what is the length of the journey? *Ans.* $111\frac{6}{7}$ miles.

7. What number multiplied by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{15}{2}$ will give a product of $3\frac{1}{8}$? *Ans.* $1\frac{2}{3}$.

8. What cost $15\frac{2}{3}$ yards of cloth at \$$2\frac{5}{8}$ per yard? *Ans.* \$$41\frac{1}{8}$.

9. A man owning $\frac{5}{8}$ of a farm sold $\frac{2}{3}$ of his share; what part did he sell, and what part had he left? *Ans.* He sold $\frac{5}{12}$ and had left $\frac{5}{24}$.

10. James is 12 years old, and $\frac{3}{4}$ of his age is 9 years less than that of his brother; how old is his brother?
Ans. 18 years.

11. If $8\frac{1}{3}$ yards of muslin at 9 cents a yard is worth $\frac{5}{6}$ of a gallon of syrup, what is a gallon of syrup worth?
Ans. 90 cents.

12. If $\frac{2}{7}$ of a yard of linen at 70 cts. a yard costs as much as $\frac{4}{5}$ of a yard of muslin, what will a yard of muslin cost? *Ans.* 25 cents.

13. If a man build $\frac{3}{5}$ of a rod of wall in a day, how many rods will he build in $11\frac{1}{3}$ days?
Ans. $6\frac{4}{5}$ rods.

14. A and B bought a barrel of flour; A paid $\$3\frac{1}{8}$, and B $\$4\frac{1}{8}$; what part should each get?
Ans. A $\frac{25}{58}$, and B $\frac{33}{58}$.

15. In an orchard $\frac{1}{3}$ of the trees bear apples, $\frac{1}{4}$ peaches, $\frac{1}{5}$ plums, and 26 are pear trees; how many trees in the orchard? *Ans.* 120 trees.

16. One-fourth of a pole is in the mud, $\frac{1}{5}$ in the water, and 22 feet above the water; what is the length of the pole? *Ans.* 40 feet.

17. What is the difference between $\frac{3}{8}$ of $\frac{2}{15}$ and $\frac{4}{9}$ of $\frac{3}{16}$?
Ans. $\frac{1}{30}$.

18. A owns $\frac{1}{2}$ of a gold mine, and B $\frac{1}{3}$ of the same; A sells $\frac{2}{3}$ of his share and B $\frac{1}{2}$ of his to C; what is now the part of each? *Ans.* A has $\frac{1}{6}$, B $\frac{1}{6}$, and C $\frac{1}{2}$.

19. A, B, and C purchased 160 acres of land; A is to have $\frac{1}{4}$ of it, which is $\frac{2}{3}$ as much as B's, and C the balance; how much is C's part? *Ans.* 60 acres.

20. My horse is worth \$150, which is $\frac{2}{3}$ the value of the buggy; what is the buggy worth? *Ans.* \$225.

21. A can do a piece of work in 4 days and B in 5 days; how much will A and B do in one day? How long will it take both to do it?

Ans. They will do $\frac{9}{20}$ in 1 day, and in $2\frac{2}{9}$ days the whole.

22. A can do a piece of work in 12 days, and B in 15 days; how long will it take both to do it?

Ans. $6\frac{2}{3}$ days.

23. A cistern has two pipes for supply and one for discharge; by the first it would be filled in 6 hours, and by the second in 8 hours, and the third would empty it in 4 hours; in what time will it be filled if all run together?

Ans. In 24 hours.

24. How does multiplying the numerator of a fraction affect its value?

25. How does dividing the numerator of a fraction affect its value?

26. How does multiplying the denominator of a fraction affect its value?

27. How does dividing the denominator of a fraction affect its value?

28. What effect has multiplying both terms of a fraction by the same number?

29. What effect has dividing both terms of a fraction by the same number?

30. To what term in division does the numerator correspond? the denominator? the fraction itself?

4 31. A can do a piece of work in 7 days; B in 9 days; how much will each do in 1 day? how much will both do in a day, and how long will it take both to do it?

Ans. A will do $\frac{1}{7}$ and B $\frac{1}{9}$ in a day; both will do $\frac{16}{63}$ in a day, and it will take both $3\frac{15}{16}$ days.

32. A man who had purchased a house, found that after he had paid $\frac{1}{2}$ and $\frac{1}{3}$ of the cost he still owed \$500; what was the cost of the house? *Ans.* \$3,000.

33. A, B, and C purchase a farm; A pays $\frac{3}{11}$, B $\frac{3}{10}$, and C the balance, which was \$4,700; how much did A and B each pay? *Ans.* A, \$3,000, B, \$3,300.

34. A and B together have \$170, and $\frac{2}{3}$ of A's money is equal to $\frac{3}{4}$ of B's; how much has each?

$$\text{A's} \times \frac{2}{3} = \text{B's} \times \frac{3}{4}$$

$$\text{A's} \times \frac{8}{12} = \text{B's} \times \frac{9}{12}$$

8 A's = 9 B's; hence A has \$9 as often as B has \$8.

Ans. A, \$90; B, \$80.

35. The sum of two numbers is 56, and $\frac{1}{2}$ of the first is equal to $\frac{2}{3}$ of the second; what are the numbers?

Ans. First, 32; second, 24.

36. The product of two numbers is $33\frac{1}{3}$, and one of the numbers is $5\frac{1}{2}$; what is the other number? *Ans.* $6\frac{2}{33}$.

37. The factors are $25\frac{3}{4}$ and $37\frac{2}{3}$; what is the product?

Ans. $969\frac{11}{12}$.

38. At \$$5\frac{2}{3}$ a ton, how many tons of coal can be bought for \$$123\frac{3}{4}$? *Ans.* $21\frac{57}{68}$ tons.

39. The weight of 3 loads of hay were: 1st, 2 T. 5 cwt. 30 lbs.; the 2d, 3 T. 2 cwt. 25 lbs. and the 3d, 2 T. 8 cwt. 60 lbs.; what did the three loads weigh?

Ans. 7 T. 16 cwt. 15 lbs. or 7.8075, or $7\frac{323}{400}$ tons.

40. The dividend is $205\frac{4}{5}$ and the quotient $25\frac{1}{4}$; what is the divisor? *Ans.* $8\frac{76}{505}$.

41. My horse is worth $1\frac{1}{2}$ times as much as my carriage; both are worth \$250; what is each worth?

Ans. Horse, \$150; carriage, \$100.

42. If $1\frac{3}{4}$ acres of land cost \$$87\frac{1}{2}$, what will $12\frac{3}{5}$ acres cost? *Ans.* \$630.

DECIMAL FRACTIONS.

Fractions whose denominators are 10, 100, 1000, etc., are rendered decimals of the same name by a little change in form; thus, a decimal point is placed on the left of the decimals, or on the right of the units, and the same relation exists between the successive orders, as in abstract numbers, but the orders themselves are reversed.

$\frac{1}{10} = .1,\qquad \frac{1}{1000} = .001,$

$\frac{1}{100} = .01,\qquad \frac{1}{10000} = .0001,$

and are read alike; thus,

one tenth, one thousandth,
one hundredth, one ten-thousandth.

Also, $\frac{3}{10} = .3$, read three tenths;
$\frac{7}{100} = .07$, seven hundredths;
$\frac{36}{100} = .36$, thirty-six hundredths;
$\frac{456}{1000} = .456$, four hundred and fifty-six thousandths.

Hence, to enumerate a decimal fraction, read it as you would an integral number, adding to this the name of the denominator, when a common fraction, which will be expressed by 1 with as many zeros attached to it as there are numbers of decimal figures.

ADDITION AND SUBTRACTION.

EXAMPLES

1. Add .1, .01, .001, .0001, and .00001; thus:

(1.)		(2.)		(3.)
.1	Add	.0234	Add	5.634
.01		.213		21.321
.001		.3146		.654
.0001		.32		.012
.00001		.6		5.364
.11111		1.4710		32.985

	(4.)		(5.)
From	4.36215	*From*	.326159
Take	1.83754	*Take*	.234573
Rem.	2.52461	*Rem.*	.091586

Cor.—As the relation of the orders are the same, and the decimals rise in value in the same direction, whilst in name they take the opposite direction; hence, addition and subtraction of decimals are performed as in Integral Numbers.

MULTIPLICATION.

THEOREM.

In the multiplication of decimals, the product will have as many places of decimals as both factors.

$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100} \quad \therefore \quad .1 \times .1 = .01,$

and

$\frac{1}{100} \times \frac{1}{10} = \frac{1}{1000} \quad \therefore \quad .01 \times .1 = .001.$

	1ST COL.		2D COL.
or,	$1 \times 1 = 1$	and	$1 \times .1 = .1$
	$.1 \times 1 = .1$	and	$.1 \times .1 = .01$
	$.01 \times 1 = .01$	and	$.01 \times .1 = .001$

The first column of products is the same as the first column of multiplicands, as 1 is the multiplier. The multiplier in the second case is one-tenth, consequently the products of the second column must be one-tenth of the first.

Therefore the product of two decimal factors will have as many decimal places as both factors.

$1 \times 1 = 1$, units.
$.1 \times .1 = .01$, hundredths.
$.01 \times .01 = .0001$, ten thousandths.

$1 \times 1 = 1$, units.
$10 \times 10 = 100$, hundreds.
$100 \times 100 = 10000$, ten thousands.

Rem.—Observe the correspondence in name, when the contrary orders are multiplied.

PROBLEMS.

1. Multiply	2. Multiply
3.156	.534
.215	.136
15780	3204
3156	1602
6312	534
.678540	.072624

Rem.—Each product must have six decimals, hence in the second example a zero must be prefixed.

ADDITION.

As the relations of the orders in United States money is the same as in abstract numbers, hence their application is the same; and in addition and subtraction like orders must be placed under each other, and in every other way the same methods are followed.

PROBLEMS.

1. What is the sum of twenty-five dollars, thirty-six cents and five mills; twelve dollars, eighteen cents and four mills; nine dollars and ten cents; thirty dollars and five mills; fifteen dollars and three cents.

$25.365
12.184
9.100
30.005
15.030
$91.684

Ans., Ninety-one dollars, sixty-eight cents and four mills.

2. Add the following sums of money:

Five dollars, thirty cents and four mills.	$5.304
Three dollars and two mills	3.002
Two dollars and three cents	2.030
Seven dollars and three mills	7.003
Twelve dollars and one cent	12.010
Nine dollars	9.000
	$38.349

(3.)	(4.)	(5.)
Add $97.548	Add $386.946	Add 287,642 mills
68.754	5372.875	548,753
97.632	64759.654	659,864
198.564	876943.687	3,217,634
$462.498	$947463.162	4,713,893 mills.

REM. 1.—The sum of the last example may be numerated thus: Four millions seven hundred and thirteen thousand, eight hundred and ninety-three mills; or, thus: Four thousand seven hundred and thirteen dollars, eighty-nine cents and three mills.

REM. 2.—Mills are numerated the same as abstract numbers.

SUBTRACTION.

$287.304
194.293
$93.011

1. From two hundred and eighty-seven dollars, thirty cents and four mills, take one hundred and ninety-four dollars, twenty-nine cents and three mills. Remainder, Ninety-three dollars, one cent and one mill.

(2.)	(3.)	(4.)
$475648.364	$9,486,397.213	$21795.375
387654.875	6,397,423.875	10963.625
$87993.489	$3,088,973.338	$10831.750

(5.)	(6.)	(7.)	(8.)	(9.)
100000	100	100	100.00	100.00
99999	99	1	1.50	2.50
1	1	99	98.50	97.50

REM.—As in addition and subtraction, so also in multiplication, the process is the same as that of abstract integers and decimals; hence there is no need of further exemplification.

English money is reckoned in pounds, shillings, pence, and farthings; sometimes also in guineas; thus,

TABLE.

4 farthings (*far.*) = 1 penny (*d.*).
12 pence = 1 shilling (*s.*).
20 shillings = 1 pound (£).
21 shillings = 1 guinea.

PROBLEMS.

Reduce £1 to shillings, pence, and farthings.

£1
20
20 = shillings.
12
240 = pence.
4
960 = farthings.

£1 = 20 shillings.
£1 = 240 pence.
£1 = 960 farthings.

As there are twenty shillings in one pound, there will always be twenty times as many shillings as pounds; and as there are twelve pence in every shilling, there will be twelve times as many pence as shillings; and four times as many farthings as pence.

Cor.—A higher denomination is reduced to a lower one by multiplication.

Reduce 960 farthings to pence, shillings and pounds; thus,

4) 960 farthings.
12) 240 pence.
20) 20 shillings.
1 pound.

As four farthings make one penny, there will be one-fourth as many pence as farthings, one-twelfth as many shillings as pence, and one-twentieth as many pounds as shillings.

COR.—A lower denomination is reduced to a higher one by division.

Reduce 1095 farthings to pence, shillings, and pounds.

4) 1095 farthings.
12) 273 . . . 3 far.
20) 22 . . . 9d.
£1 2s. 9d. 3 far.

The first remainder is farthings, the second pence, and the third shillings.

Reduce £1 2s. 9d. 3 far. to farthings.
20
22 shillings.
12
273 pence.
4
1095 farthings.

In reducing a higher denomination to a lower one, begin by multiplying by the number of the next lower denomination that makes one of the higher, and if it be a compound number, add to the product the number of the lower denomination, and continue this process until you reach the lowest denomination.

In reducing a lower to a higher denomination, divide by the number of the lowest denomination that makes one of the next higher, and if there be a remainder, it will be of the lowest denomination, etc.

COR.—In the computation of compound numbers, instead of carrying a unit to a higher order for every ten, as in abstract numbers, a unit is carried to a higher denomination as often as the sum reaches the number that it takes of the lower denomination to make one of the next higher denomination; thus, as 4 farthings make 1 penny, as often as the sum of the farthings reaches four, one must be carried to the pence; and as 12 pence make 1 shilling, in computing pence as many must be carried to shillings as the number of times 12 is contained in the number of pence; 1 from shillings to pounds for every 20.

In division, the order is reversed, as then we begin with the highest denomination and descend.

EXAMPLES.

	£	*s.*	*d.*	*far.*
1. Add	3	8	7	3
	5	9	6	2
	6	11	9	1
	8	15	11	3
	24	5	11	1

The sum of the first column is 9 farthings, which is 2 times 4 and 1; the 1 is farthings, and must be placed under the farthings; the 2 is carried to the next denomination and added with the pence, the sum of which is 35; that is, 2 times 12 and 11, that is, 2 shillings and 11 pence; the 2 is added with the shillings, making the sum 45, which is £2 5s.; the shillings are placed under the shillings and the 2 carried to the pounds, the sum of which is 24.

2 0

/ 8

	£	s.	d.	far.
2. From	54	6	5	1
	28	7	6	3
	£25	18s.	10d.	2 far.

As you cannot subtract 3 farthings from 1 farthing, you must borrow 1 penny, which is 4 farthings; this 4 and the 1 make 5; then 3 from 5, 2 remains; the 1 penny borrowed must be carried to the 6, which makes 7, which cannot be subtracted from 5; 1 shilling, that is, 12 pence, must be borrowed and added to the 5, which makes 17; 7 from 17, 10 remains; 1 shilling to carry to 7 makes 8, which cannot be taken from 6; 1 pound, that is, 20 shillings, must be borrowed and added to the 6, making 26, from which subtract 8 and 18 remains; and £1 to carry to 28, making 29, which is subtracted from 54 and 25 remains.

Rem.—When the subtrahend is less than the minuend, the difference can be taken directly.

	£	s.	d.	far.
3. Multiply	4	6	5	3
by				5
	£21	12s.	4	3

4	6	5	3
5	5	5	5
21	20) 32	25	) 15
	£1 12s.	3	3d. 3 far.
		12) 28	
		2s. 4d.	

Cor.—Multiply each denominate number, and divide the product by the number of this denomination that it takes to make one of the higher, and carry the number of times it is contained to the higher denomination, and place the remainder under its kind.

4. Multiply £48 12s. 7d. 2 far. by 6.

```
5. Divide     4 ) £5   6s.   3d.   1 far.   by 4.
                  £1   6s.   6d.   3¼ far.
```

4 is contained in 5, once and £1 over; this £1 is 20 shillings, which added to the 6 shillings make 26 shillings, into which 4 is contained 6 times and 2 shillings over; this 2 shillings is 24 pence, which added to the 3 pence, makes 27 pence, in which 4 is contained 6 times and 3 pence over, which is 12 farthings, and 1 more make 13, in which 4 is contained 3¼ times.

6. Divide £754 15s. 9d. 3 far. by 27.

```
27 ) £754  15s.  9d.  3 far. ( £27
      54
     ---
     214
     189
     ---
      25
       20
     ----
      515 ( 19s.   Add the 15s.
      27
      ---
      245
      243
      ---
        2
       12
      ---
       33 ( 1d.   Add the 9d.
       27
       --
        6
        4
       --
       27 ( 1 far.   Add the 3 far.
       27
       --
Quotient = £27 19s. 1d. 1 far.
```

7. Multiply £5 4s. 6d. 1 far. by 35.

$$\begin{array}{rrrr} \pounds 5 & 4\text{s.} & 6\text{d.} & 1\text{ far.} \\ & & & 35 \\ \hline \pounds 182 & 18\text{s.} & 2\text{d.} & 3\text{ far.} \end{array}$$

$$\begin{array}{r} 35 \\ \underline{5} \\ 175 \\ \underline{7} \\ \pounds 182 \end{array} \qquad \begin{array}{r} 35 \\ \underline{4} \\ 140 \\ \underline{18} \\ 20)\,158\,(7 \\ \underline{140} \\ 18 \end{array} \qquad \begin{array}{r} 35 \\ \underline{6} \\ 210 \\ \underline{8} \\ 12)\,\underline{218} \\ 18\text{s. } 2\text{d.} \end{array} \qquad \begin{array}{l} 4)\,\underline{35} \\ \quad 8\text{d. } 3\text{ far.} \end{array}$$

Rem.—Observe these solutions carefully; for if they are understood, there is no further difficulty in denominate numbers; the principle is the same in all, the tables alone differ.

EXAMPLES.

1. In 2 dollars, how many cents? How many mills?

$$\begin{aligned} \$2 \times 100 &= 200 \text{ cents.} \\ 2 \times 1000 &= 2000 \text{ mills.} \end{aligned}$$

2. In 5 dollars, how many cents? How many mills?

3. In 7 dollars, how many cents? How many mills?

4. In 5 dollars 15 cents, how many cents? How many mills?

$$\begin{array}{l} \$5 = 500 \text{ cents.} \\ \qquad\ \ \underline{15} \\ \qquad 515 \text{ cents} = 5150 \text{ mills.} \end{array}$$

5. In 6 dollars 15 cents and 3 mills, how many mills?

6. In 500 cents, how many dollars? $\frac{500}{100} = \$5$, *Ans.*

7. In 625 cents, how many dollars and cents?

Ans. $6.25.

8. In 5325 mills, how many dollars, cents, and mills?
Ans. $5.325.

9. In 63257 mills, how many dollars, cents, and mills?

10. In 75325 cents, how many dollars and cents?

11. If 1 bushel of wheat cost $1.125, what will 8 bushels cost?

12. If 1 bushel of wheat cost $1.05, what will 10 bushels cost?

13. If 1 bushel of wheat cost $1.05, what will 100 bushels cost?

14. If 8 bushels of wheat cost $9, what cost 1 bushel?

15. If 8 bushels of wheat cost $9, what cost 35 bushels?

16. If 10 bushels of wheat cost $10.50, what cost 53 bushels?

17. Bought dry goods for $243.37; groceries for $146.294; hardware for $71.96; notions for $21.512. What was the amount of the bill? Sold the same at a profit of $157.192. What did I sell the whole for?

18. If 5 lbs. sugar cost 50 cents, what will 6 lbs. cost? 7 lbs.? 8? 9? 10? 11? 12?

19. If 6 lbs. cost 72 cts., what will 7 lbs. cost? 8 lbs.? 9? 10? 11? 12?

20. In 15 farthings, how many pence?
Ans. 3¾ pen

21. In 18 farthings, how many pence? How many pence in 21 far.? 23? 25? 27? 29? 31? 33? 34? 35?

22. How many shillings in 25 pence? in 28? 35? 38? 45? 51? 56? 65?

23. How many pounds in 35 shillings? in 40? 50? 60? 65? 70? 75? 80? 85? 90? 95? 100? 105? 110? 120?

24. How many farthings in £9 13s. 9d. 3 far.?

25. How many pounds, shillings, pence, and farthings in 37864321 farthings?

26. Multiply £4 8s. 9d. 3 far. by 9.

27. Divide £25 9s. 4d. 1 far. by 13?

AVOIRDUPOIS, OR COMMERCIAL WEIGHT,

is used in commercial transactions, when goods are bought or sold in quantity, and for all metals except gold and silver.

TABLE.

16 drams (*dr.*) = 1 ounce (*oz.*)
16 ounces = 1 pound (*lb.*).
25 pounds = 1 quarter (*qr.*).
4 quarters = 1 hundredweight (*cwt.*).
20 cwt. = 1 ton (*T.*).

EXEMPLIFICATION.

$$\begin{array}{r l}
1 & \text{T.} \\
\underline{20} & \\
20 & \text{cwt.} \\
\underline{4} & \\
80 & \text{qrs.} \\
\underline{25} & \\
2000 & \text{lbs.} \\
\underline{16} & \\
32000 & \text{oz.} \\
\underline{16} & \\
512000 & \text{dr.}
\end{array}
\qquad\qquad
\begin{array}{r l}
16\,\underline{)\,512000} & \text{dr.} \\
16\,\underline{)\,32000} & \text{oz.} \\
25\,\underline{)\,2000} & \text{lbs.} \\
4\,\underline{)\,80} & \text{qrs.} \\
20\,\underline{)\,20} & \text{cwt.} \\
1 & \text{T.}
\end{array}$$

T. cwt. qr. lb. oz. dr.
Reduce 3 4 2 8 6 10
20
64
4
258
25
6458
16
103334
16
1653354 drams.

16) 1653354
16) 103334 . . . 10 dr.
25) 6458 . . . 6 oz.
4) 258 . . . 8 lb.
20) 64 . . . 2 qr.
3 4 2 8 6 10
T. cwt. qr. lb. oz. dr.

Reduce 1653354 drams to the original denominations.

TROY WEIGHT

is used for gold, silver, and jewels; also in philosophical experiments.

TABLE.

24 grains (*gr.*) = 1 pennyweight (*pwt.*)
20 pennyweights = 1 ounce.
12 ounces = 1 pound.

1 lb.
12
12 oz.
20
240 pwt.
24
5760 qr.

24) 5760 gr.
20) 240 pwt.
12) 12 oz.
1 lb.

Reduce 5 lb. 6 oz. 10 pwt. 16 gr.
12
66 Add the 6 oz.
20
1330 Add the 10 pwt.
24
31936 Add the 16 gr.

Reduce 31936 grs. to the original denominations.

24) 31936 gr.
20) 1330 . . . 16 gr.
12) 66 . . . 10 pwt.
5 lb. 6 oz. 10 pwt. 16 gr.

DIAMOND WEIGHT.

Used for diamonds and other precious stones.

TABLE.

16 parts = 1 grain = .8 grain Troy.
4 grains = 1 carat = 3.2 grains Troy.

APOTHECARIES' WEIGHT

is used by druggists in putting up prescriptions; the pound, ounce, and grain are the same as in Troy Weight.

TABLE.

20 grains = 1 scruple (℈).
3 scruples = 1 dram (ʒ).
8 drams = 1 ounce (℥).
12 ounces = 1 pound.

$$\begin{array}{r l}
1 & \text{lb.} \\
\underline{12} & \\
12 & ℥ \\
\underline{8} & \\
96 & ʒ \\
\underline{3} & \\
288 & ℈ \\
\underline{20} & \\
5760 & \text{gr.}
\end{array}
\qquad\qquad
\begin{array}{r l}
20\,)\,\underline{5760} & \text{gr.} \\
3\,)\,\underline{288} & ℈ \\
8\,)\,\underline{96} & ʒ \\
12\,)\,\underline{12} & ℥ \\
1 & \text{lb.}
\end{array}$$

APOTHECARIES FLUID WEIGHT

is used for liquids in medical prescriptions.

TABLE.

60 minims (♏) = 1 fluid dram (f ʒ).
8 fluid drams = 1 fluid ounce (f ℥).
16 fluid ounces = 1 pint (O.).
8 pints = 1 gallon (*Cong.*).

For ordinary use, 1 teacup = 2 wine glasses = 8 table-spoons = 32 tea-spoons = 4 f ℥.

COMPARISON OF WEIGHTS.

1 lb. Avoirdupois = 7000 gr. Troy.
1 lb. Troy = 5760 gr. Troy.

LINEAR MEASURE

is used for lengths and distances.

TABLE.

12 inches (*in.*) = 1 foot (*ft.*).
3 feet = 1 yard (*yd.*).
5½ yds., or 16½ ft. = 1 rod (*rd.*).
40 rods = 1 furlong (*fur.*).
8 furlongs = 1 mile (*m.*).
3 miles = 1 league (*lea.*).

MARINER'S MEASURE.

6 feet = 1 fathom.
120 fathoms = 1 cable length.
880 fath., or 7⅓ cable lengths = 1 mile.

REM.—1 nautical league = 3 equatorial miles = 3.45771 statute miles. 60 equatorial miles = 69.1542 statute miles = 1 equatorial degree (°). 360° = the circumference of a circle. 360 equatorial degrees = the circumference of the earth.

CLOTH MEASURE.

2¼ inches (*in.*) = 1 nail (*na.*).
4 nails or 9 in. = 1 quarter.
4 quarters = 1 yard.
3 quarters = 1 ell Flemish.
5 quarters = 1 ell English.
6 quarters = 1 ell French.

SURVEYOR'S MEASURE OF LENGTH.

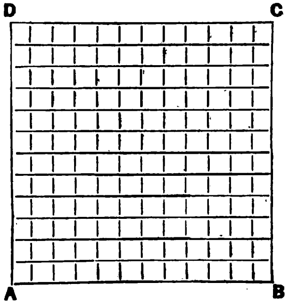
$7\frac{92}{100}$ inches = 1 link (*l.*).
25 links = 1 pole (*p.*).
100 links, 4 poles, 66 feet = 1 chain (*ch.*).
10 chains = 1 furlong.
8 furlongs, or 80 chains = 1 mile.

LAND MEASURE.

40 perches = 1 rod.
4 rods = 1 acre.
640 acres = 1 square mile,
termed a Section.

SQUARE MEASURE.

A ***Square*** is a surface bounded by four equal sides, its angles are also equal; thus,



This figure, ABCD, represents a square foot, considering each of the small spaces as an inch. The sides, AB, BC, AD, and DC, each equal to 12 inches in length. The angles, A, B, C, and D, are equal; that is, if one is placed on the other, the sides respectively will coincide.

AB is 12 inches long, and every inch in width makes 12 square inches; and the 12 inches in width, which is either AD or BC, makes 12 × 12 = 144 sq. in.; hence the correspondence of linear and square measure; thus,

LINEAR MEASURE.

12 inches = 1 foot.
3 feet = 1 yard.
5½ yards = 1 rod, pole, or perch.

SQUARE MEASURE.

12 × 12 = 144 square inches = 1 square foot.
3 × 3 = 9 square feet = 1 square yard.
5½ × 5½ = 30¼ square yards = 1 square rod;
also called perch.

CUBIC MEASURE.

12 × 12 × 12 = 1728 cubic inches = 1 cubic foot.
3 × 3 × 3 = 27 cubic feet = 1 cubic yard.

CUBIC MEASURE.

A ***Cube*** is a solid figure bounded by six equal squares; the square on page 72 represents the base or any other side, as the sides are all equal; the length of all the edges are equal, and the angles are all equal. If the above figure have 12 inches altitude added to it, every inch will make 144 cubic inches and 12 inches in altitude, 144 × 12 = 1728, which is the number of cubic inches in a cubic foot.

TABLE.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
16 cubic feet = 1 cord foot.
128 cubic feet, or 8 cord feet } = 1 cd. of wood, bark, etc.

40 cubic feet of round timber, or 50 cubic feet of hewn timber = 1 Ton.

A perch of stone is 16½ feet long, 1½ feet wide, and 1 ft. high = 24¾ solid feet.

LIQUID MEASURE.

This measure is used for all liquids.

TABLE.

4 gills (*gi.*) = 1 pint (*pt.*).
2 pints = 1 quart (*qt.*).
4 quarts = 1 gallon (*gal.*).

In all liquids, except ale, beer, and milk, the gallon is 231 cubic inches.

In ale, beer, and milk, it is 282 cubic inches.

Rem.—In the former 31½ gallons is called a barrel, 63 gallons a hogshead, 42 gallons a tierce, 84 gallons a puncheon, and 126 gallons a pipe, and 2 pipes a tun. In the latter, 36 gallons = a barrel, and 54 gallons = a hogshead; these, however, are not measures, but only vessels.

DRY MEASURE

is used for grain, fruits, vegetables, coal, salt, etc.

TABLE.

2 pints = 1 quart.
8 quarts = 1 peck (*pk.*).
4 pecks = 1 bushel (*bu.*)
= 2150.42 cubic inches.

The wine gallon of United States = 231 cu. in.
The beer gallon of United States = 282 cu. in.
The dry gallon of United States = 268.8 cu. in.
Imperial gal. of Great Britain for dry and liquid measures = 277.274 cu. in.
Dry bushel of United States = 2150.42 cu. in.
Imperial bushel of Great Britain = 2218.192 cu. in.

TIME TABLE.

60 seconds (*sec.*) = 1 minute (*m.*).
60 minutes = 1 hour (*hr.*).
24 hours = 1 day (*da.*).
7 days = 1 week (*wk.*).
30 days = 1 month (*mo.*).
365 days = 1 common year.
366 days = 1 leap year.

ANGULAR OR CIRCULAR MEASURE

is applied to angles and circumferences, reckoning latitudes and longitudes, etc.

TABLE.

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
30 degrees = 1 sign (*S.*).
12 signs or 360 degrees = 1 circumference.

Apparently the sun makes an entire revolution of the earth in 24 hours,* and consequently travels 15° in 1 hour; therefore,

1 hour of time = 15° longitude.
1 minute of time = 15′ longitude.
1 second of time = 15″ longitude.

MISCELLANEOUS TABLE.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.
24 sheets of paper = 1 quire.
20 quires = 1 ream.
196 lbs. = 1 barrel of flour.
200 lbs. = 1 barrel of pork.

When a sheet of paper is folded into two leaves, or 4 pages, and a book made in this way, it is called a folio.

4 leaves is called a quarto.
8 leaves is called an octavo.
12 leaves is called a duodecimo.

* *Really,* the revolution is that of the earth on its own axis.

PRACTICAL QUESTIONS.

1. Reduce £3 9s. 11d. 3 far. to farthings.

2. Reduce £12 15s. 8d. to pence.

3. Reduce £7 0s. 2d. to pence.

4. Reduce 2354 farthings to the higher denominations.

5. Reduce 543 pence to the higher denominations.

6. Reduce 731 shillings to the higher denominations.

7. Reduce 3 T. 6 cwt. 2 qr. 12 lb. 6 oz. and 9 dr. to drams.

8. Reduce 672432 drams to the higher denominations.

9. Reduce 5 lb. 8 oz. 9 pwt. 15 gr. to grains.

10. Reduce 64324 grains Troy to the higher denominations.

11. Reduce 2 lb. 6 ℥ 4 ʒ 2 ℈ 10 gr. to grains.

12. Reduce 6742 gr., Apothecaries weight, to the higher denominations.

13. Reduce 3 lea. 2 mi. 5 fur. 24 rods 2 yd. 1 ft. 6 in. to inches.

14. Reduce 802456 inches to the higher denominations.

15. Reduce 4 yards 3 qrs. 2 na. and 2 inches to inches.

16. Reduce 5 ells Flemish to inches.

17. Reduce 4 ells English to inches.

18. Reduce 3 ells French to inches.

19. Reduce 4 sq. rods 8 sq. yd. 105 sq. ft. and 112 sq. in. to square inches.

20. Reduce 3 cu. yd. 12 cu. ft. and 1236 cu. in. to cubic inches.

21. A pile of wood is 16 ft. long, 4 ft. high, and the length of the wood is 4 feet. How many cords of wood?

$$\frac{16 \times 4 \times 4}{8 \times 4 \times 4}$$

22. Reduce 25 gal. 3 qt. 1 pt. and 3 gills to gills.

23. Reduce 9 bu. 3 pk. 4 qt. 1 pt. to pints.

24. How many years, months, and days, from April 15th, 1842, to June 20th, 1850?

yr.	mo.	da.
1850	6	20
1842	4	15
8	2	5

25. How many years, months, and days, from October 25th, 1845, to August 18th, 1850?

yr.	mo.	da.
1850	8	18
1845	10	25
4	9	23

REM.—In this question, instead of using the eighth and tenth months, some authors prefer calling them 7 months and 9 months; but if there is an inaccuracy in the months, there is also in the years; hence if we read the above thus: The one thousand eight hundred and fiftieth year, the 8th month and 25th day, there is no inaccuracy.

In computations of time we always take 30 days as a month.

26. The difference in time of two places is 2 hr. 2 min. and 2 sec.; what is the difference in longitude?

hr.	min.	sec.
2	2	2
		15
30°	30′	30″

Ans. Thirty degrees, 30 minutes, and 30 seconds.

52. Bought 480 yds. cloth at 66⅔ cts., and sold it at 87½ cts.; what was the profit?

$$\tfrac{1}{8} \text{ of } 480 = \$60; \quad \tfrac{7}{8} = \$420$$
$$\tfrac{1}{3} \text{ of } 480 = 160; \quad \tfrac{2}{3} = \underline{\ 320\ }$$
$$\$100, \text{ Profit.}$$

53. Bought 480 yds. at 37½ cts., and sold it for 50 cts. per yard; what was the profit?

$$\tfrac{1}{2} \text{ of } 480 = \$240$$
$$\tfrac{1}{8} \text{ of } 480 = 60; \quad \tfrac{3}{8} = \underline{\ 180\ }$$
$$\$60, \text{ Profit.}$$

54. Bought 600 yds. cloth at $1 per yard, and sold it for $1.25 per yard; what was the profit?

$$600 \times 1\tfrac{1}{4} = \$750$$
$$600 \times 1 = \underline{\ 600\ }$$
$$\$150, \text{ Profit.}$$

55. How many yards of cloth, at 12½ cts. per yard, can be bought for $240? 8 yds. can be bought for every dollar.

$$240 \times 8 = 1920 \text{ yds.}$$

56. How many yards at 16⅔ cts.? 20 cts.? 25 cts.? 37½ cts.? 50 cts.? 62½ cts.? 75 cts.? 87½ cts.?

$$37\tfrac{1}{2} = \tfrac{3}{8}; \quad \overset{80}{\cancel{240}} \times \tfrac{8}{3} = 640 \text{ yds.};$$
$$\text{for } 62\tfrac{1}{2} \text{ cts.} = 240 \times \tfrac{8}{5}.$$

57. Sold 500 barrels flour at $6.62½ per barrel, and invested the proceeds in different kinds of dry goods, averaging 87½ cts. per yard. What were the proceeds of the flour, and how many yards of goods did I get?

58. How many years, months, and days from January 21st, 1874, to April 30th, 1876?
Ans. 2 y. 3 m. 9 days.

59. From June 18th, 1865, to April 15th, 1869?
Ans. 3 y. 9 m. 27 d.

60. From Dec. 30th, 1872, to Jan'y 5th, 1875?
Ans. 2y. 0 m. 5 d.

61. The difference in longitude of two places is 15°; what is the difference in time? *Ans.* 1 hour.

62. The difference in longitude of New York and Cincinnati is 10° 35′; what is the difference in time?
Ans. 42 min. 20 sec.

63. The difference in longitude of New York and St. Louis is 16° 14′; what is the difference in time?
Ans. 1 h. 4 min. 56 sec.

64. The difference in longitude of Philadelphia and Cincinnati is 9° 20′; what is the difference in time?
Ans. 37 min. 20 sec.

65. The difference in the time of London and Washington is 5 h. 8 m. 4 sec.; what is the difference in longitude? *Ans.* 77° 1′.

66. When it is noon at New York, what is the time 15° east of New York? 15° west?
Ans. 1 P.M. and 11 A.M.

67. When it is noon at Cincinnati, what is the time at New York? *Ans.* 12 h. 42 min. 20 sec. P.M.

68. When it is noon at New York, what is the time at Cincinnati? *Ans.* 11 h. 17 min. 40 sec. A.M.

69. When it is noon at New York, what is the time at St. Louis? *Ans.* 1 h. 4 min. 56 sec. P.M.

70. When it is noon at St. Louis, what is the time at New York? *Ans.* 10 h. 55 min. 4 sec. A.M.

99. How many pint bottles can be filled from $31\frac{1}{2}$ gal. cider? *Ans.* 252 bottles.

100. What is the value of a pile of wood 32 ft. long, 12 feet high, and 4 feet wide, at \$5 a cord? *Ans.* \$60.

101. If a man earn \$9.25 a week, how much will he earn in 5 weeks? *Ans.* \$46.25.

102. A man earned $\$95.37\frac{1}{2}$, and spent $\$34.87\frac{1}{2}$; how much had he left? *Ans.* \$60.50.

103. Bought $37\frac{3}{4}$ yards of cloth at \$5.25 per yard; what was the cost? *Ans.* $\$198.18\frac{3}{4}$.

104. What is the cost of $257\frac{3}{4}$ acres of land at \$37.45 per acre? *Ans.* $\$9,652.73\frac{3}{4}$.

105. What is the cost of $542\frac{1}{4}$ bushels of wheat at $\$1.12\frac{1}{2}$ per bushel? *Ans.* $\$610.03\frac{1}{8}$.

106. What is the product of $\frac{37}{250}$ and .625? *Ans.* $\frac{37}{400}$, or .0925.

107. What part of 5 gals. 3 qts. 1 pt. is 3 gal. 2 qt. 1 pt.? *Ans.* $\frac{29}{47}$.

DEF. The reciprocal of a number is formed by inverting the number thus: 2, 3, 4 may be written $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$; their reciprocals are $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$; so also of any fraction— $\frac{2}{3}$, $\frac{3}{4}$, $\frac{6}{5}$, the reciprocals $\frac{3}{2}$, $\frac{4}{3}$, $\frac{5}{6}$.

108. What is the value of any number multiplied by its reciprocal? *Ans.* 1.

109. How much alloy must be mixed with 2 lb. 8 oz. pure gold to make it 18 carats fine?

Pure gold is 24 carats fine. *Ans.* 10 oz. 13 pwt. 8 gr.

110. If 4 equal angles are made at the center of a circle, how many degrees in each? *Ans.* 90 degrees.

These angles are termed right angles.

3 1/2

RATIO.

Two fractions can be formed with any two integral numbers, the one a proper fraction and the other an improper fraction; thus, $\frac{7}{9}$ and $\frac{9}{7}$ can be formed with 7 and 9. When the proper fraction is a multiplier of any number, the product is less than the number multiplied; therefore, this fraction is termed a ***Diminishing Ratio.*** But when the improper fraction is a multiplier, the product is greater than the multiplicand; hence, the improper fraction is termed an ***Increasing Ratio.***

PROBLEMS.

1. If 5 lbs. sugar cost 50 cents, what will 9 lbs. cost?

It is evident that 9 lbs. will cost more than 5, and just as much more as is indicated by the increasing ratio formed by the two like terms, 5 lbs. and 9 lbs.

If 5 lbs. cost 50 cts., 9 lbs. will cost $\overset{10}{\cancel{50}}$ cts. $\times \frac{9}{\cancel{5}} = 90$ cts.; this may be further demonstrated thus,

5 lbs. = 50 cts.
1 lb. = 10 cts.
9 lbs. = 90 cts.

In a problem of ratios, the one ratio is given, and one of the terms of the other ratio, to get the second term; thus, in the above:

Given, 5 lbs. sugar and 50 cents.
Required, 9 lbs. sugar and ?

The ratio of the money will be the same as of the sugar. As the required sugar is more than the given, the ratio must be increasing; that is, $\frac{9}{5}$. $\therefore$ $50 \times \frac{9}{5} = 90$, the ratio of the required money to the given, $\frac{90}{50} = \frac{9}{5}$, the same as of the sugar.

EXAMPLES.

1. If 5 bushels of wheat cost \$6.25, what will 8 bushels cost?

Given 5 bu. and \$6.25.
Required, 8 bu. and

$$\$\cancel{6}.\cancel{2}\cancel{5} \times \frac{8}{\cancel{5}} = \$10.00.$$

$$\frac{8}{5} = 25)\frac{1000}{625} = 5)\frac{40}{25} = \frac{8}{5}.$$

The ratios of the wheat and of the money is the same.

Rem.—Ratios can only be formed by two like terms.

2. If 5 bushels of oats cost \$1.50, what will 21 bushels cost?

$\frac{21}{5} = \frac{630}{150} = \frac{21}{5}$; the ratio is the same.

Given 5 bu. and \$1.50.
Required, 21 bu. and

$$\$\cancel{1}.\cancel{5}\cancel{0} \times \frac{21}{\cancel{5}} = \$6.30.$$

Rem.—Write the given terms in a line and the like term of the required immediately under that of the given. One term of the required is wanting, and the given like term may be called the term of demand, and should be placed first and multiplied by the ratio, having for its numerator the required term of the ratio, and for its denominator the given term.

As a general thing, an increase in the required term of the ratio will take more of the unknown to accomplish it; an increased amount of goods will cost a greater sum of money; an enlarged piece of work, an additional sum of money; and the greater the work, the longer time to perform it, etc. In examples of this kind, the ratios are direct, and the required term of the ratio holds the place of the numerator and the given term that of the denominator, and the product of the ratio and the odd given term is the term required.

3. If a man travel 40 miles in 8 hours, how many miles will he travel at that rate in 18 hours?

Given	40 miles	and	8 hours.
Required,	? miles	and	18 hours.

$$\overset{5}{\cancel{40}} \text{ miles} \times \frac{18}{\cancel{8}} = 90 \text{ miles.}$$

4. If 15 bushels of wheat yield 3 barrels of flour, how many bushels will yield 10 barrels of flour?

Given	15 bu.	and	3 barrels of flour.
Required,	?	and	10 barrels of flour.

SOLUTION.

$$\overset{5}{\cancel{15}} \times \frac{10}{\cancel{3}} = 50 \text{ bushels.}$$

5. If a man travel 30 miles in 2 days, how long will it take him to travel 240 miles?

Given	30 miles	and	2 days.
Required,	240 miles	and	?

6. If a staff 4 feet long cast a shadow 3 feet, what is the height of a steeple which casts a shadow 90 feet?

	STAFF.	SHADOW.
Given	4 ft.	3 ft.
Required,	?	90 ft.

7. If the interest of $100 for one year is $5, what would be the interest of $500 for the same time?

	PRINCIPAL.		INTEREST.
Given	$100	and	$5.
Required,	$500	and	?

8. If $\frac{2}{3}$ of a barrel of flour cost $4, what will $4\frac{2}{3}$ barrels cost?

Given	$\frac{2}{3}$ barrel	and	$4.
Required	$4\frac{2}{3}$ barrel	and	?

$$4 \times \frac{4\frac{2}{3}}{\frac{2}{3}}. \qquad \$4 \times \frac{\overset{7}{\cancel{14}}}{\cancel{3}} \times \frac{\cancel{3}}{\cancel{2}} = \$28.$$

Rem. $4\frac{2}{3}$ is a multiplier, and $\frac{2}{3}$ is a divisor; the $\frac{2}{3}$ must be inverted.

9. If $4\frac{1}{2}$ bushels of wheat cost $5.40, what will $8\frac{3}{4}$ bu. cost? $9\frac{1}{2}$ bu.? $23\frac{3}{4}$? $31\frac{2}{3}$? $47\frac{1}{2}$? $39\frac{1}{6}$? $58\frac{3}{4}$? $97\frac{1}{2}$? $106\frac{3}{4}$?

10. If 8 bushels of wheat cost $10, what will be the cost of $9\frac{1}{2}$ bu.? $10\frac{1}{4}$ bu.? $15\frac{3}{5}$ bu.? $37\frac{1}{5}$ bu.? $95\frac{3}{5}$ bu.? $125\frac{3}{5}$ bu.? $150\frac{4}{5}$ bu.? $279\frac{2}{5}$ bu.?

11. If $5\frac{3}{4}$ acres of land cost $230, what is the cost of $6\frac{1}{2}$ acres? $7\frac{3}{4}$ acres? $12\frac{1}{2}$ acres? $13\frac{1}{8}$ acres? $17\frac{2}{8}$? $18\frac{3}{8}$? $19\frac{4}{8}$? $20\frac{5}{8}$? $37\frac{6}{8}$? $49\frac{7}{8}$?

12. If $2\frac{1}{5}$ acres of land cost $110, what will $\frac{3}{4}$ of an acre cost? $\frac{1}{2}$ acre? $\frac{1}{4}$ acre? $\frac{1}{5}$ acre? $\frac{2}{5}$ acre? $\frac{3}{5}$ acre? $\frac{4}{5}$ acre? $1\frac{1}{5}$ acres? $1\frac{2}{5}$ acres?

13. If $\frac{5}{12}$ of a yard of cloth cost $\frac{9}{10}$ of a dollar, what will $\frac{2}{7}$ of a yard cost?

Given	$\frac{5}{12}$ yd.	and	$\$\frac{9}{10}$.
Required,	$\frac{2}{7}$ yd.	and	?

$$\frac{9}{10} \times \frac{2}{7} \times \frac{12}{5}.$$

14. If ½ yard of cloth cost $2, what will 3 ells Fl. cost?

$$2 \times \tfrac{9}{2} =$$

15. If ¾ yard of cloth cost $2.25, what will 5 ells English cost?

$$2.25 \times \tfrac{25}{3}.$$

What will 5 ells French cost?

In the preceding examples, the ratios were all direct; as in those cases any increase in the required term of the ratio demanded a similar increase of the unknown; but there are cases which require the ratio to be inverted, such as, the more men employed, the less time will be required to perform a piece of work; the more hours employed in the day, the less days; the wider the material, the less yards it will take to make a garment, etc.

These cases of inverse ratio are readily detected by asking this question: "Will an increase of the required term of the ratio demand an increase in the unknown term?" If it does, the ratio is direct; but if an increase in the required term of the ratio demand a diminution of the unknown term, the ratio must be inverted; thus,

PROBLEM.

If 4 men can do a piece of work in 10 days, how long will it take 8 men to do the same work?

Given 4 men in 10 days.
Required, 8 men in ?

$$\overset{5}{\cancel{10}} \text{ days} \times \frac{\cancel{4}}{\underset{2}{\cancel{8}}} = 5 \text{ days.}$$

Cor.—It is evident that 8 men will do it in less time; that is, in one-half the time that it will take 4, which ratio is expressed by the diminishing ratio of 4 and 8, that is, $\frac{4}{8} = \frac{1}{2}$, in which the given term is the numerator of the ratio and the required term the denominator.

EXAMPLES.

1. If 5 men can dig a ditch in 20 days, how many men will dig it in 25 days?

Given	5 men	and	20 days.
Required,	?	and	25 days.

$$\cancel{5}\text{ men} \times \frac{\overset{4}{\cancel{20}}}{\underset{1}{\cancel{25}}} = 4\text{ men.}$$

Rem.—An increase in the days will require less men.

2. If 6 horses eat a certain quantity of hay in 30 weeks, how many horses will consume the same quantity of hay in 9 weeks?

Given	6 horses	and	30 weeks.
Required,	?	and	9 weeks.

$$\overset{2}{\cancel{6}}\text{ horses} \times \frac{\overset{10}{\cancel{30}}}{\underset{1}{\cancel{9}}} = 20\text{ horses.}$$

3. If a man perform a journey in 12 days, when the days are 9 hours long, how many days of 12 hours will it take him?

Given	12 days	and	9 hours.
Required,	?	and	12 hours.

4. How many yards of lining $\frac{3}{4}$ yd. wide will it take to line 3 yards of cloth $\frac{6}{4}$ yd. wide?

Given $\frac{6}{4}$ yard wide and 3 yards long.
Required, $\frac{3}{4}$ yard wide and ?

$$\cancel{3} \times \frac{6}{\cancel{3}} = 6 \text{ yds.}$$

COMPOUND RATIO.

When there are two or more ratios, it is termed ***Compound Ratio;*** thus,

If 3 men in 12 days build 40 rods of wall, how many rods will 9 men build in 24 days?

Given 3 men, 12 days, 40 rods.
9 men, 24 days, ?

$$40 \times \frac{\overset{\cancel{3}}{\cancel{9}}}{\cancel{3}} \times \frac{\overset{6}{\cancel{24}}}{\underset{\cancel{4}}{\cancel{12}}} = 240 \text{ rods.}$$

Rem.—Each ratio is direct.

If 12 men dig a ditch 20 rods long in 18 days by working 8 hours a day, how many men will dig a ditch 40 rods long in 24 days, working 6 hours a day?

Given 12 men, 20 rods, 18 days, 8 hours.
? 40 rods, 24 days, 6 hours.

$$12 \text{ men} \times \frac{\overset{2}{\cancel{40}}}{\cancel{20}} \times \frac{\overset{\cancel{3}}{\cancel{18}}}{\underset{\cancel{4}}{\cancel{24}}} \times \frac{\cancel{8}}{\cancel{6}} = 24 \text{ men.}$$

Exemplification.—The longer the trench, the more men it will take, and the ratio is direct; but the greater the number of days and the more hours of each day, the less men would be required; hence these two ratios are inverse.

Cor.—Each ratio must be dealt with as in the preceding article.

EXAMPLES.

1. If 3 men in 8 days of 9 hours each build a wall 20 ft. long, 2 ft. thick, and 4 ft. high, in how many days of 8 hours each will 12 men build a wall 100 ft. long, 3 ft. wide, and 6 ft. high?

Given 3 men, 8 days, 9 hrs., 20 ft. l., 2 ft. w., 4 ft. h.
12 men, ? 8 hrs., 100 ft. l., 3 ft. w., 6 ft. h.

$$\cancel{8}\text{ days} \times \frac{\cancel{3}}{\cancel{12}} \times \frac{9}{\cancel{8}} \times \frac{\overset{5}{\cancel{100}}}{\cancel{20}} \times \frac{3}{\cancel{2}} \times \frac{\overset{3}{\cancel{6}}}{4} = \frac{9 \times 5 \times 3 \times 3}{4 \times 4}$$

$$= \frac{405}{16} = 25\tfrac{5}{16}\text{ days.}$$

Rem.—The men and hours are inverse, the other ratios direct.

2. If 6 men mow 12 acres of grass in 2 days of 10 hrs. each, how many hours a day must 8 men work to mow 40 acres in 4 days? *Ans.* 12½ hours a day.

3. If 6 horses eat 36 bushels of oats in 18 days, how many bushels will be sufficient for 12 horses 24 days? *Ans.* 96 bushels.

4. If $100 in 12 months gain $6, how long will it take $500 to gain $15?

Given $100 prin., 12 mo., $6 int.
Required, $500 prin., ? $15 int.

$$\overset{2}{\cancel{12}}\text{ mo.} \times \frac{1}{\cancel{5}} \times \frac{\overset{3}{\cancel{15}}}{\cancel{6}} = 6\text{ months.}$$

5. If 6 men manufacture 300 pairs of shoes in 30 days, how many men will make 900 pairs of shoes in 60 days? *Ans.* 9 men.

PERCENTAGE.

Per Cent. means per hundred, and is generally expressed fractionally; thus, 5 per cent., 6 per cent., marked 5% and 6%, is expressed $\frac{5}{100}$, $\frac{6}{100}$, etc., or .05, .06; thus, $\frac{5}{100}$ of 100 $= \cancel{100} \times \frac{5}{\cancel{100}} = 5$, and $\frac{6}{100}$ of 100 is 6.

EXAMPLES.

1. What is 5% of 200? $\overset{2}{\cancel{200}} \times \frac{5}{\cancel{100}} = 10$, *Ans.*

What is 5% of 300? *Ans.* 15.

What is 5% of 400? *Ans.* 20.

2. What is 5% of 245? $2.45 \times \frac{5}{\cancel{100}} = 12.25$, *Ans.*

The 100 is canceled in the 245 by pointing off two places of decimals.

3. What is 6% of 300? $\overset{3}{\cancel{300}} \times \frac{6}{\cancel{100}} = 18$, *Ans.*

What is 6% of 400? *Ans.* 24.

What is 6% of 500? *Ans.* 30.

4. What is 6% of 368? $3.68 \times \frac{6}{\cancel{100}} = 22.08$.

COMMISSION, OR BROKERAGE.

The business of a commission merchant or broker is to make purchases and sales, on which he receives a percentage.

PROBLEM I.

A purchase of $100 worth of goods, at 1% commission, will cost $101; that is, $\frac{101}{100}$ of the amount of the purchase.

4. A broker makes sales for $4325, at 2%; what is the brokerage, and what does the owner realize?

$$43.25 \times \frac{2}{\cancel{100}} = \$86.50, \text{ commission.}$$

$$43.25 \times \frac{98}{\cancel{100}} = \$4238.50, \text{ owner realized.}$$

5. A merchant sells to a broker $3275 uncurrent funds at 5% discount; what does he realize?

$$\overset{163.75}{\$\cancel{3275}} \times \underset{20}{\frac{\overset{19}{\cancel{95}}}{\cancel{100}}} = \$3111.25.$$

6. An architect charges 1½% for plans and specifications, and 2½% for superintending a building, the cost of which is $10000; what is the architect's fees? *Ans.* $400.

7. A broker has 2% commission, and 3% for guaranteeing payment; what does he receive on sales amounting to $42325? *Ans.* $2116.25.

8. I sent my broker $4000 to purchase goods; what amount of goods did he purchase after deducting commissions at 2% on the amount of goods? $4000 × $\frac{100}{102}$.

For every $102 he gets $100 worth of goods.

9. Bought a draft on New York, the face of it $500 premium, ¼%; what is the whole cost and the premium?

$$\$500 \times \frac{\cancel{100}\frac{1}{4}}{\cancel{100}} \times \frac{401}{400} = \qquad \$501.25$$

$$\begin{array}{rr} & 500.00 \\ \hline \text{Premium,} & \$1.25 \end{array}$$

10. Sold goods to the amount of $4444, and invested the proceeds, after retaining my commissions, which were 2% on the sales, and 1% on the investment; what was the amt. of investment?

$$\overset{44}{\$\cancel{4444}} \times \frac{98}{\cancel{100}} \times \frac{\cancel{100}}{\cancel{101}} = \$4312.$$

INTEREST.

Interest is an allowance for the use of money. It is reckoned by percentage; thus, 5%, 6%, etc., meaning for a year, when not otherwise expressed; for any other time it is as the ratio of the time; thus, the interest of $100 at 6% is $6 for a year, for two years $12, and for six months $3.

PROBLEMS.

1. Find the interest of $150, at 6%, for 1 year.

$$\$150 \times \tfrac{6}{100} = \$9.$$

For 8 months.

$$\$1.50 \times \frac{\cancel{6}}{\cancel{100}} \times \frac{\overset{4}{\cancel{8}}}{\underset{2}{\cancel{12}}} = 1.50 \times \frac{4}{\cancel{100}} = \$6.00.$$

For 6 months.

$$\$1.50 \times \frac{\cancel{6}}{\cancel{100}} \times \frac{\overset{3}{\cancel{6}}}{\underset{2}{\cancel{12}}} = \$4.50.$$

For 14 months.

$$\$1.50 \times \frac{\cancel{6}}{\cancel{100}} \times \frac{\overset{7}{\cancel{14}}}{\underset{2}{\cancel{12}}} = 1.50 \times \frac{7}{\cancel{100}} = \$10.50.$$

Cor.—At 6%, the rate per cent. for any number of months is ½ the number of months; thus, for 8 months it is 4%, for 6 months it is 3%, and for 14 months 7%.

2. Find the interest of $150, at 6%, for 129 days?

$$\$150 \times \frac{\cancel{6}}{100} \times \frac{129}{\underset{60}{\cancel{360}}} = \$150 \times \frac{129}{6000} = \$3.225.$$

Cor.—The interest of a sum of money for any number of days is equal to the product of the sum of money and

5. A note dated 1st April, 1870, for $200, bearing interest from date at 6%, has the following endorsements on the back of it.

May 1st, 1871, paid $12.
May 1st, 1872, paid $12.
May 1st, 1873, paid $12.

What was due on the note April 1st, 1875, when it was paid?

Int. of $200 for 1 year is	$12
	5 years.
	$60
Add Principal,	200
	$260
Deduct sum of payments,	36
April 1st, 1875, balance,	$224 paid.

As the sum of the payments never equaled the interest, no computation need be made until the end, when the sum of the payments must be deducted.

Prob. 1.—In the case of Partial Payments, when the payment is greater than the interest until the time of the payment, the interest is computed until the date of the payment and added to the principal, and from this sum the payment is deducted, and the balance is regarded as the principal of the note.

Prob. 2.—When the payment is less than the interest, no computation is made; but whenever the sum of the payments is greater than the interest until that time, then the interest is to be computed to the date and added to the principal, and from this sum the sum of all the payments is deducted, and the balance regarded as the principal of the note.

These computations are to be repeated until the note is paid.

BANK DISCOUNT.

Bank Discount is reckoned the same as interest on the face of the note and for the time the note is given, plus three days, which are called days of grace, and the note need not be paid until the last day of grace, three days after the time specified in the note.

It is called Discount, because the borrower does not receive the sum specified in the note, but the difference of this sum and the interest.

Notes in Banks are usually given for a short time, viz., for 30 days, 60 days, or 90 days; and the interest is computed for 33 days, 63 days, or 93 days.

1. The bank discount on a note for $100 at 30 days.

$$\$\cancel{100} \times \frac{\cancel{6}}{\cancel{100}} \times \frac{33}{\underset{60}{\cancel{360}}} = \frac{11}{20} = \$.55.$$

Borrower gets $100 — $.55 = $99.45.

2. Borrower gets $100—$1.05 = $98.95. $100 for 60 da.

$$\$\cancel{100} \times \frac{\cancel{6}}{\cancel{100}} \times \frac{63}{\underset{60}{\cancel{360}}} = \frac{21}{20} = \$1.05.$$

3. Borrower gets $100—1.55=$98.45. $100 for 90 days.

$$\$\cancel{100} \times \frac{\cancel{6}}{\cancel{100}} \times \frac{93}{\underset{60}{\cancel{360}}} = \frac{31}{20} = \$1.55.$$

4. Borrower gets $324—$5.02 = $318.98. $324 for 90 days.

$$\$\overset{162}{\cancel{324}} \times \frac{\overset{31}{\cancel{93}}}{\underset{1000}{\cancel{6000}}} = \$5.022.$$

TRUE DISCOUNT

is the abatement made on a note not yet due, or the difference between the face of the note and a sum of money which, placed at interest, will amount to the face of the note by the time the note is due; thus, the present value of a note for $100, due in one year without interest, when money is worth 6%, is such a sum as will amount to $100 in one year at 6% interest.

COR.—As $100 now will be worth $106 at the end of the year, so the present worth of money due in one year is $\frac{100}{106}$ of the face of the note due in one year.

COR.—The ratio to obtain the present worth from the face of the note, has for its numerator 100, and the denominator is 100 increased by the interest for the time and rate.

EXAMPLES.

1. What is the present worth of a note for $324, due in one year, rate of money 6%?

$$\$324 \times \tfrac{100}{106} = \$305.66+, \text{ present worth.}$$

$$\$18.34, \text{ discount.}$$

2. Due in 2 years 1 month.

$$\text{Rate} = \frac{100}{112\frac{1}{2}} = \frac{200}{225} = \frac{8}{9}.$$

$$\text{9 months} = \frac{100}{104\frac{1}{2}} = \frac{200}{209}$$

EXCHANGE.

Def. 1.—***Exchange*** is the system by which payments are made at a distant place by means of Bills of Exchange or Drafts.

2. The person making or signing the Bill or Draft, is called the ***Maker*** or ***Drawer;*** the one to whom it is addressed is the ***Drawee,*** and the person to whom it is ordered to be paid is the ***Payee.***

3. The person in possession of the draft is the ***Holder,*** and if he endorse it, he becomes responsible for the payment, unless otherwise specified.

4. Exchange between the different cities of one's own country is ***Domestic,*** and that with a foreign country is ***Foreign Exchange.***

DOMESTIC EXCHANGE.

PROBLEM.

To find the cost of a draft on Philadelphia or New York when at a premium, and also when at a discount; thus, if the premium is 1%, the draft will cost $\frac{101}{100}$ of its face; if at a discount of 1%, it will cost $\frac{99}{100}$ of its face.

Rem.—The face of the draft must be equal to the sum of money that we wish to remit.

EXAMPLES.

1. What is the cost of a draft on Philadelphia for $500, at ½% premium?

$$500 \times \frac{100\frac{1}{2}}{100} = 5\cancel{00} \times \frac{201}{2\cancel{00}} = \frac{1005}{2} = 502\tfrac{1}{2}.$$

2. I owe $3000 in New York, and the premium on a New York draft is ¼%. What must I pay for the draft?

$$\overset{15}{\$\cancel{3000}} \times \frac{100\frac{1}{4}}{100} = \frac{401}{400} = 15 \times \frac{401}{2} = \$3007.50.$$

3. What is the cost of a draft on New Orleans for $500, at ½% discount?

$$5\cancel{00} \times \frac{99\frac{1}{2}}{1\cancel{00}} = 497\tfrac{1}{2}.$$

Rem.—The computation may be made as interest; thus, first example, $500 at 1% is $5, and ½ of $5 is $2½ = premium; second example, $3000 at 1% is $30, at ¼% it is $7.50 premium; in the third example, $500 at ½% is $2.50, the discount.

5. What is the cost of a draft, at 60 days, on New York for $500, premium ½%, interest off at 6%?

$$\begin{aligned} \text{Discount} &= \tfrac{63}{60}\% = \tfrac{21}{20}\%. \\ \text{Premium} &= \tfrac{1}{2}\% = \tfrac{10}{20}\%. \\ \text{Discount above premium} &= \tfrac{11}{20}\%. \end{aligned}$$

$$\$\cancel{500} \times \frac{11}{\underset{4}{\cancel{2000}}} = \$2.75, \text{ discount.}$$

$$\begin{array}{r} \$500 \\ 2.75 \\ \hline \$497.25, \text{ cost of draft.} \end{array}$$

£ 500

FOREIGN EXCHANGE.

EXAMPLES.

1. What is the cost in New York of a draft on London for £500, exchange at \$4.80 to the £?

$$£500 \times 4.80 = \$2400.$$

2. What amount of debt in London can be paid with \$4000 in New York, rate of exchange as above?

$$\frac{\$4000.0}{\$4.8} = \frac{10000}{12} = £833\ 6\text{s. } 8\text{d.}$$

3. What will a draft on London for £480 15s. 6d. cost in New York, rate as above?

$$\begin{array}{c} 15\text{s.} \quad 6\text{d.} \\ \underline{12} \\ \frac{186}{240} = \frac{31}{40}. \end{array}$$

$$£480\tfrac{31}{40} \times 4.80 = \$2307.72.$$

4. What will \$500 in New York pay in Paris, exchange on London as above, and £1 = 25.2 Francs.

$$\overset{125}{\$\cancel{500}} \times \frac{\overset{\overset{21}{\cancel{63}}}{\cancel{25.2}}}{\underset{\underset{\cancel{4}}{\cancel{12}}}{\cancel{4.8}}} = 125 \times 21 = 2625 \text{ Francs.}$$

5. What sum in New York will pay 2625 Francs in Paris?

6. What will \$2540 in New York pay in Paris? as above.

7. What sum in New York will pay 52500 Francs in Paris, exchange at 25.2 Francs to the £ and £1 = \$4.80?

Rem.—As the rates of exchange are not constant, it is not proper to insert a table.

AVERAGING ACCOUNTS.

When sales are made at different times and on different terms, to find a mean time when the whole may be paid without loss to either party; thus,

A merchant sells goods as follows:

January 1st for \$100 on 1 month.
January 18th for \$200 on 1 month.
February 1st for \$300 on 2 months.
February 12th for \$250 on 3 months.

No payment has yet been made. In how many days should the whole be paid, in order that there be no loss to either party?

Begin when the first account is due, February 1st.

$$\begin{array}{rcrcrl}
\$100 & \times & 0 & = & & \\
200 & \times & 17 & = & 3400 & \\
300 & \times & 59 & = & 17700 & \\
\underline{250} & \times & 100 & = & \underline{25000} & \\
850 & & & 850\,) & 46100 & (\text{ 54 days.} \\
 & & & & \underline{4250} & \\
 & & & & 3600 & \\
 & & & & \underline{3400} & \\
 & & & & 200 & = \frac{200}{850} = \frac{4}{17}.
\end{array}$$

Due 54 days after February 1st, that is, 27th March.

BANK ACCOUNT.

In bank, accounts are closed at the end of the year, and the balance, which is always in favor of the depositor, is brought to the credit side of the account, as over-drawing is not permitted.

To this balance each deposit is added at the date on which it is made, and from the credit side is subtracted each sum drawn by check at its date.

Each balance, or sum, is multiplied by the number of days from its date until the next transaction, and lastly by the time between the last transaction and the end of the year.

The sum of all these products will be the number of days that one dollar is at interest.

DR. JOHN JOHNSON in acct. with MECHANICS' BANK. CR.

1877.			1876.			DA.	PRO.
Jan. 6.	To check.	$200	Dec. 31.	By bal. old acct.	$600	6	3600
" 15.	" "	100	Jan. 10.	By cash $300.	400	4	1600
" 18.	" "	200	Jan. 25.	By cash $750.	700	5	3500
" 29.	" "	350			600	3	1800
					400	7	2800
					1150	4	4600
					800	2	1600
						6)	19,500

Interest, 3.25

The last sum or difference on the credit side is the Principal.

$800
Interest = 3.25
Total balance = $803.25

The account is only rendered for one month.

ALLIGATION.

Alligation is the mixing of different qualities of grain, groceries, liquors, etc., either to obtain an average price of the mixture, which process is termed ***Alligation Medial,*** or to make a mixture of several kinds that shall have a certain value, and this is called ***Alligation Alternate.***

ALLIGATION MEDIAL.

EXAMPLES.

1. Mix together 10 lbs. of tea at 40 cents a lb., 12 lbs. at 50 cts. a lb., 5 lbs. at 60 cts. per lb., and 3 lbs. at $1.00 per lb.; what is the mixture worth per lb.?

lbs.		*cts.*		*cts.*
10	×	40	=	400
12	×	50	=	600
5	×	60	=	300
3	×	100	=	300
30 lbs.			=	1600 cts.
1 lb.			=	53⅓ cts.

2. Mix together 12 gallons of wine at 50 cents a gal., 10 gallons at 60 cents, 5 gallons at 80 cents, and 3 gallons of water; what is the mixture worth per gallon?

$$\begin{array}{rcrcl}
\textit{gal.} & & \textit{cts.} & & \textit{cts.}\\
12 & \times & 50 & = & 600\\
10 & \times & 60 & = & 600\\
5 & \times & 80 & = & 400\\
\underline{3} & \times & 0 & = & \underline{0}\\
30 \text{ gal.} & & & = & 1600 \text{ cts.}\\
1 \text{ gal.} & & & = & 53\tfrac{1}{3} \text{ cts. per gal.}
\end{array}$$

5. Mix 10 gallons of wine worth \$2 a gallon, 8 gallons at \$1.50, 16 gallons at \$1.25, and 4 gallons of water; what is one gallon worth?

$$\begin{array}{rcrcr}
10 & \times & 2 & = & \$20\\
8 & \times & 1.50 & = & 12\\
16 & \times & 1.25 & = & 20\\
\underline{6} & \times & 0 & = & \underline{0}\\
40 \text{ gal.} & & & = & \$52\\
1 \text{ gal.} & & & = & \$1.30
\end{array}$$

6. Mix 50 lbs. coffee worth 14 cents, 40 lbs. at 20 cts., 100 lbs. at 15 cts., and 30 lbs. at 10 cts.; what is a lb. of the mixture worth? *Ans.* 15 cts.

7. A farmer has 10 pigs worth \$3 each, 12 worth \$4 each, and 8 large ones worth \$9 each; what is the average worth? *Ans.* \$5 each.

ALLIGATION ALTERNATE.

PROBLEM.

What relative quantity of each must be taken of two kinds of sugar, the one worth 5 cts. per lb. and the other 9 cts., in order that the mixture be worth 6 cts. per lb.?

$$6\left\{\begin{array}{l|l|l} 5 & 1 & 3 \\ 9 & \frac{1}{3} & 1 \end{array}\right.$$

$= 15$

$\frac{1}{4}$ lb. $= \frac{9}{24}$

1 lb. $= 6$

By making 1 lb. at 5 cents worth 6 cents, there is a gain of 1 cent; and in 1 lb. at 9 cents made at 6 cents, there is a loss of 3 cents; hence, in $\frac{1}{3}$ of a lb. at 9 cents there is a loss of 1 cent; therefore, in mixing 1 lb. at 5 cents with $\frac{1}{3}$ of a lb. at 9 cents, the mixture is worth 6 cents, and the gain and loss are equal. By reducing the 1 and $\frac{1}{3}$ to a common denominator and multiplying both by it, they become integral numbers, and the relative quantity of each is the same.

Cor. 1.—*It matters not how many different qualities are to be mixed, they must be mixed by pairs, one of the values less and the other greater than the mean value.*

Cor. 2.—*If there are odd numbers to be mixed, or more above or below the mean value, one of those having the smaller number must be mixed with two or more of those having the greater number on the other side of the mean.*

EXAMPLES.

1. Mix coffee at 15 cts., 17, 19, and 22 cts. per lb., so that the mixture is worth 18 cts.

$$8\left\{\begin{array}{l|l|l} 15 & \frac{1}{3} & 1 \\ 19 & 1 & 3 \end{array}\right. \qquad 18\left\{\begin{array}{l|l|l} 17 & 1 & 4 \\ 22 & \frac{1}{4} & 1 \end{array}\right. \qquad 18\left\{\begin{array}{l|l|l} 15 & \frac{1}{3} & 4 \\ 22 & \frac{1}{4} & 3 \end{array}\right. \qquad 18\left\{\begin{array}{l|l} 17 & 1 \\ 19 & 1 \end{array}\right.$$

$$\left\{\begin{array}{l|l|l|l} 15 & \frac{1}{3} & 1 & 4 \\ 17 & 1 & 4 & 1 \\ 19 & 1 & 3 & 1 \\ 22 & \frac{1}{4} & 1 & 3 \end{array}\right. \qquad 18\left\{\begin{array}{l|l|l} 15 & \frac{1}{3} & 1 \\ 17 & 1 & 4 \\ 19 & 1 & 3 \\ 22 & \frac{1}{4} & \end{array}\right.$$

The sum of these will also prove a correct mixture; that is,

$$\begin{array}{rclcr}
5 & \text{at} & 15 & = & 75 \\
5 & \text{at} & 17 & = & 85 \\
4 & \text{at} & 19 & = & 76 \\
4 & \text{at} & 22 & = & \underline{88} \\
 & & & 18\,) & \underline{324} \\
 & & & & 18
\end{array}$$

Cor.—If any two values, one greater and the other less, be connected together, and the difference between each one and the mean value be placed opposite the one with which it is connected, the quantity opposite each value will be its relative quantity.

Rem.—The relative quantity of each is not absolute, as any two opposite quantities may be mixed, and, after being mixed differently, these mixtures may be added and the relative quantities changed.

2. A grocer has spices at 18, 24, 36, and 42 cts. per lb., of which he wishes to make a mixture worth 32 cents per lb.

$$32\left\{\begin{array}{l}18\\24\\36\\42\end{array}\right.\quad
\begin{array}{rcrl}
10 & = & 180 & \\
4 & = & 96 & \\
8 & = & 288 & \\
\underline{14} & = & \underline{588} & \\
36 &) & 1152 & (\ 32\text{ cts.} \\
 & & \underline{108} & \\
 & & 72 & \\
 & & \underline{72} &
\end{array}$$

3. A merchant has five qualities of liquors, at the following prices per gallon, viz., \$1.25, \$1.45, \$1.60, \$1.80,

and $1.90; and he has an order for liquor at $1.50 per gallon; at the rate of $1.90 he has but 40 gallons, all of which he wishes to put in the mixture; how much must be taken of each of the other kinds?

150	$1.25	10 + 40 =	$50 \times \frac{8}{5} = 80$
	1.45	30	$30 \times \frac{8}{5} = 48$
	1.60	25	$25 \times \frac{8}{5} = 40$
	1.80	5	$5 \times \frac{8}{5} = 8$
	1.90	25	$25 \times \frac{8}{5} = 40$

$$25 \times \frac{8}{5} = 40.$$

4. A farmer has six kinds of wheat of different values per bushel, viz., $1.00, $1.05, $1.10, $1.20, $1.35, and $1.40; he makes a sale at $1.15 per bushel; how many bushels of each kind must he take?

115	$1.00	25	=	2500
	1.05	20	=	2100
	1.10	5	=	550
	1.20	5	=	600
	1.35	10	=	1350
	1.40	15	=	2100
		80		) 92.00
				1.15, Proof.

NOTE.—Different connections will produce different results; the only principle involved is that of the two quantities mixed, one must be less and the other greater than the mean.

INVOLUTION AND EVOLUTION.

Involution and ***Evolution*** correspond very nearly to multiplication and division. Involution consists in the multiplication of the same number by itself, or of the same factor employed two or more times to produce the product; whilst Evolution restores the common factors from the product.

INVOLUTION

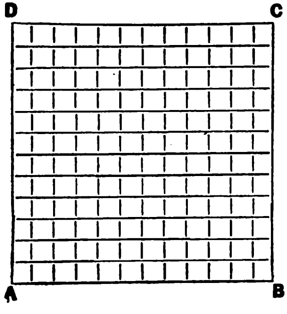
is called the raising of powers; thus, $2 \times 2 = 4$, is called the second power of 2, and is expressed by 2^2.

$2 \times 2 \times 2 = 8 =$ 3d power of 2, and is expressed by 2^3.

$3 \times 3 = 9 =$ 2d power of 3, and is expressed by 3^2.

$3 \times 3 \times 3 = 27 =$ 3d power of 3, and is expressed by 3^3.

THEOREM.

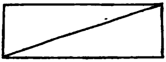


The area of a square is equal to the square of its side.

As there are 12 inches in the length of this square, and for every inch in breadth there will be 12 sq. in., thus, $12 \times 1 = 12$ sq. in.; $12 \times 2 = 24$ sq. in.; $12 \times 5 = 60$ sq. in.; and $12 \times 12 = 144$.

The area of the square is equal to the square of its side.

Cor. 1.—If, instead of taking the whole side for its breadth, a part only is taken, as 1 in., 2 in., 3 in., or 5 in., the figure is a rectangle, and its area is the product of its length and breadth.



Cor. 2.—If a diagonal is drawn to the rectangle, it divides it into two equal triangles.

Cor. 3.—It is proved by a simple geometrical demonstration that the area of any parallelogram is also equal to the product of its base and perpendicular altitude; therefore the area of any triangle is equal to one-half the product of its base and perpendicular altitude.

Rem. 1.—A plumb-line which extends directly toward the centre of the earth is apparently perpendicular to the surface of the earth, but really perpendicular to a plane which is a tangent to the surface of the earth.

Rem. 2.—The floor of a house is either tangent to the surface of the earth, or is parallel to another plane which is tangent to the surface of the earth.*

Rem. 3.—Planes holding these positions are called horizontal planes, and planes which are everywhere equally distant are parallel.

EVOLUTION,

or the extracting of roots, is exactly the reverse of involution, and is expressed by the radical sign $\sqrt{\ }$ for square root, and $\sqrt[3]{\ }$ for cube root.

As we have seen in square measure that the area of the surface is found by the product of two equal sides, so in evolution we have given the surface to find the side.

---

* A line touching a circle, or a plane touching a sphere at but one point is called a *tangent.*

As the product is the result of two equal factors, it appears to be the natural solution to factor this product; thus,

PROBLEMS.

$4 = 2 \times 2$; therefore 2 is the square root of 4.
$9 = 3 \times 3$; therefore 3 is the square root of 9.
$16 = 2 \times 2 \times 2 \times 2 = 4 \times 4$; $\therefore$ 4 is the sq. root of 16.

$25 = 5 \times 5.$
$36 = 6 \times 6.$
$49 = 7 \times 7.$
$64 = 8 \times 8.$
$81 = 9 \times 9.$
$100 = 10 \times 10.$
$121 = 11 \times 11.$
$144 = 12 \times 12.$

$169 = 13 \times 13.$
$196 = 14 \times 14.$
$225 = 15 \times 15.$
$256 = 16 \times 16.$
$289 = 17 \times 17.$
$324 = 18 \times 18.$
$361 = 19 \times 19.$
$400 = 20 \times 20.$

Rem.—Above is given all the square numbers to 400.

Cor.—For any other numbers below 400 only an approximate root can be obtained.

$2 \times 2 \times 2 = 8.$ 2 is the cube root of 8.
$3 \times 3 \times 3 = 27.$ 3 is the cube root of 27.
$4 \times 4 \times 4 = 64.$ 4 is the cube root of 64, etc.

$5 \times 5 \times 5 = 125.$
$6 \times 6 \times 6 = 216.$
$7 \times 7 \times 7 = 343.$
$8 \times 8 \times 8 = 512.$

$9 \times 9 \times 9 = 729.$
$10 \times 10 \times 10 = 1000.$
$11 \times 11 \times 11 = 1331.$
$12 \times 12 \times 12 = 1728.$

Rem.—When the given number is not a power of an integer, the root may still be approximated by trial with decimals. Another method is generally adopted; thus,

You can readily see the number of figures the root is composed of.

$$\begin{array}{rcrcr}
1\times 1 = 1 & \qquad & 9\times 9 = 81 \\
11\times 11 = 1{,}21 & \qquad & 99\times 99 = 98{,}01 \\
111\times 111 = 1{,}23{,}21 & \qquad & 999\times 999 = 99{,}80{,}01
\end{array}$$

An additional figure in the root produces two in the power. When the left-hand figure in the root is small, there will be an odd number of figures in the power; but when the left-hand figure is more than 3, there will always be twice as many figures in the power as are in the root.

In raising a number consisting of two figures to the second power; as,

$$\begin{array}{r}
12 \\
\underline{12} \\
144
\end{array}$$

$$\begin{array}{rcr}
10\times 10 & = & 100 \\
2\times 10\times 2 & = & 40 \\
2\times 2 & = & \underline{4} \\
 & & 144
\end{array}$$

144 is composed of the square of 10, and of twice 10+2 multiplied by 2; twice 10+2 must therefore be the second divisor; hence,

$$\begin{array}{rll}
10\ ) & 144 & (\ 10 \\
 & \underline{100} & \\
22\ ) & 44 & (\ \underline{2} \\
 & \underline{44} & 12\text{, root.}
\end{array}$$

When there are three figures in the root; as,

$$\begin{array}{r} 123\\ 123\\ \hline 369\\ 246\\ 123\\ \hline 15129 \end{array}$$

$$\begin{array}{rcr} 100 \times 100 & = & 10000\\ 200 + 20 \times 20 & = & 4400\\ (200 + 40 + 3)\,3 & = & \underline{729} \end{array}$$

$$\begin{array}{rrl} 100\,) & \overline{15129} & (\,100\\ & 10000 & \\ 220\,) & \overline{5129} & (\quad 20\\ & 4400 & \\ 243\,) & \overline{729} & (\quad \underline{3}\\ & \underline{729} & \quad 123 \end{array} \qquad\qquad \begin{array}{rl} & 1,51,29\ (\,123\\ & 1\\ 22\,) & \overline{51}\\ & 44\\ 243\,) & \overline{729}\\ & \underline{729} \end{array}$$

Rem.—The result of the last process, without the zeros, is the same as of those illustrated with the zeros.

Cor.—To extract the square root of a number, point it off in periods of two figures, beginning with units. Place in the root the largest figure, the square of which is not greater, but is either equal to, or less than the left-hand period of the power. Square the root obtained, and subtract the square from the left-hand period; bring down the next period, and for a divisor double the root found, regarding it as tens, and try how often it will go into the power brought down; place this figure in the root, and add it to the divisor, which completes the divisor; then divide as before, and bring down the next period; again doubling the root, regarding it as tens, and continue the same process. X

SERIES OF COMMON DIFFERENCES.

A ***Series of Common Differences,*** usually termed Arithmetical Progression, is one in which the difference of any two consecutive terms, taken in order, is the same; as, in the series 1, 2, 3, 4, 5, etc., the first term is 1, and the common difference is 1; and it is an increasing series as each successive term is greater than the preceding one.

1, 3, 5, 7, etc., is an increasing series, and the common difference is 2.

2, 5, 8, 11, etc., is an increasing series, and the common difference is 3.

12, 10, 8, 6, 4, 2, etc., is a decreasing series, and the common difference is 2.

THEOREM I.

The sum of a series of equal differences is equal to one-half the product of the number of terms and the sum of the first and last terms.

As, the sum of six terms of the series, 1, 2, 3, 4, etc., is

$$\begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 \\ \hline 7 & 7 & 7 & 7 & 7 & 7 \end{array}$$

Sum of 6 terms of both series is

$$6 \times 7 = 42.$$

Sum of 6 terms of one series is

$$\tfrac{1}{2}(6 \times 7) = 21.$$

The sum of 8 terms of the series:

$$\begin{array}{rrrrrrrr}
2 & 5 & 8 & 11 & 14 & 17 & 20 & 23 \\
23 & 20 & 17 & 14 & 11 & 8 & 5 & 2 \\
\hline
25 & 25 & 25 & 25 & 25 & 25 & 25 & 25
\end{array}$$

The sum of 2 series is

$$8 \times 25 = 200$$

The sum of 1 series is

$$\tfrac{1}{2}(8 \times 25) = 100$$

Therefore the sum of a series of common differences is one-half the product of the number of terms, multiplied by the sum of the first and last terms.

THEOREM II.

Any term of an increasing series is equal to the first term plus the common difference taken as many times as the number of the terms less one; and any term of a decreasing series is equal to the first term minus the quantity that is to be added above.

The series may be written

$$\begin{array}{llllll}
 & 1 & 2 & 3 & 4 & 5 \\
 & a, & a+d, & a+2d, & a+3d, & a+4d, \text{ etc.,} \\
\text{and} & a, & a-d, & a-2d, & a-3d, & a-4d, \text{ etc.,}
\end{array}$$

in which a represents the first term and d the common difference, and in each term d is once less than the number of terms, either plus or minus; hence any term of a series of equal differences, is equal to the first term plus or minus the common difference taken as many times as the number of the term less one.

Let n represent the number of terms, and any term may be regarded as the last term, and let l represent the last term; thus,

$$l = a + (n - 1)\,d;$$

and

$$l = a - (n - 1)\,d.$$

EXAMPLES.

1. What is the sum of 25 terms of the series 1, 2, 3, 4, 5, etc.?

$$S = \tfrac{25}{2}\,(1 + 25) = 325.$$

2. What is the 10th term of the series 1, 3, 5, etc.?

$$\text{10th term} = l = a + (n - 1)\,d.$$
$$l = 1 + 18 = 19.$$

3. What is the sum of the 10 terms of the series?

$$S = \tfrac{10}{2}\,(1 + 19) = 100.$$

4. Find the 6th term of the series 12, 10, 8, etc.

$$\text{6th} = l = a - (n - 1)\,d.$$
$$l = 12 - 10 = 2.$$

5. Find the sum of six terms of the series.

$$\tfrac{6}{2}\,(12 + 2) = 42.$$

6. Find the sum of 100 terms of the series 1, 2, 3, 4, etc.

$$l = 100.$$
$$S = \tfrac{100}{2}\,(1 + 100) = 5050.$$

GENERAL REVIEW.

In the preceding pages almost every variety of Arithmetical Principles has been analyzed and developed, and quite a number of examples have been given under each head. The following Practical Examples have been selected in order to give the pupil a general review of the course. He will find it beneficial to refer back to the analyses, as he can never be successful in practice without a thorough knowledge of principles. Let this be his motto: "To gain a knowledge of the principle first, and then solve examples under the principle until it is indelibly fixed in the mind.

PRACTICAL EXAMPLES.

1. The divisor is 21, the quotient 35, and the remainder 12; what is the dividend? *Ans.* 747.

2. The three factors are 25, 36, and 42; what is the number? *Ans.* 37,800.

3. Reduce 37,800 to its prime factors. *Ans.* 5, 5, 2, 2, 3, 3, 2, 3, 7.

4. Find the greatest common divisor of 12, 18, 24, 30. *Ans.* 6.

5. Find the least common multiple of 5, 15, 18, 36. *Ans.* 180.

6. Find the least common denominator of $\frac{1}{5}$, $\frac{1}{15}$, $\frac{1}{18}$, $\frac{1}{36}$. *Ans.* 180.

7. Find the least common denominator of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$. *Ans.* 60.

8. If $\frac{2}{7}$ of a ton of coal cost $2, what will $5\frac{1}{14}$ tons cost? *Ans.* $35.50.

9. If A can dig a ditch in 9 days, B in 12 days, and C in 15 days, how long would it take the three men if they work together? *Ans.* $3\frac{39}{47}$ days.

10. A merchant bought goods for $4,675, on which he wishes to gain 20%; how much must he sell the goods for? *Ans.* $5,610.

11. In a cargo of damaged goods a merchant has goods which cost $7,420, and on which he is obliged to lose 25%; what must the sales amount to? *Ans.* $5,565.

12. A ship caught in a storm was obliged to throw overboard one-fourth of its cargo. At what rate per cent. of gain must a merchant sell his goods in order that he may neither gain nor lose by the transaction? *Ans.* $33\frac{1}{3}$%.

13. A ship on a cruise has provisions to last 1,200 men 6 months; at the end of 4 months finds that they cannot return in less than 6 more months; how many men must disembark, in order that those remaining may have provisions for that time?

Ans. 800 men must leave the vessel.

14. What cost $6\frac{3}{4}$ yards of cloth at $$2\frac{2}{3}$ a yard? *Ans.* $18.

15. At $$\frac{3}{4}$ per bushel, how many bushels of apples can be bought for $18? *Ans.* 24 bush.

16. How much land at $$37\frac{1}{2}$ per acre can be bought for $625? *Ans.* $16\frac{2}{3}$ acres.

17. The product of two factors is $\frac{5}{9}$, and one of the factors is $\frac{3}{7}$; what is the other factor? *Ans.* $\frac{35}{27}$.

18. The quotient is $\frac{1\frac{1}{2}}{5}$ and the dividend $\frac{5}{9}$; what is the divisor? *Ans.* $\frac{50}{27}$.

19. If 8 horses cost $900, what will 12 horses cost? *Ans.* $1,350.

20. If $\frac{6}{7}$ of an acre of land cost $120, what will $11\frac{1}{4}$ acres cost? *Ans.* $1,575.

21. A farmer sold to a merchant 5 cows at $24.50 each, 2 horses at $76.25 each, and 12 sheep at $4.75 each; in payment he received 28 yards of carpet at $1.25 per yard, 36 yards cloth at $4.25 per yard, and the balance in money; how much money did he receive? *Ans.* $144.

22. Washington was born 22d Feb., 1732, and died 14th Dec. 1799; what was his age? *Ans.* 67 y. 9 m. 22 d.

23. John Adams was born Oct. 19th, 1735, and died July 4th, 1826; what was his age? *Ans.* 90 y. 8 m. 15 d.

24. Thomas Jefferson was born April 2d, 1743, and died July 4th, 1826; how old was he? *Ans.* 83 y. 3 m. 2 d.

25. A note for $300, due three months after date, was given April 1st, 1875, and the principal and interest at 6% was paid Oct. 16th, 1876; what was the amount? *Ans.* $323.25.

26. If 3 men build $2\frac{1}{4}$ rods of wall in one day, how many rods will 6 men build in 8 days? *Ans.* 36 rods.

27. If 5 men in 4 days build 12 rods, how many men will build 72 rods in 15 days. *Ans.* 8 men.

28. What is the cost of collecting $24,000 taxes at $\frac{1}{2}$%? *Ans.* $120.

29. What is the premium on a bond for $3,000 at $\frac{3}{4}$%? *Ans.* $22.50.

30. What is the commission on $3,600 at $1\frac{1}{2}$%? *Ans.* $54.

31. A hare is 120 rods ahead of a greyhound, and the hare runs 8 rods while the hound runs 12; how many rods will the hound run until he overtakes the hare?

Ans. 360 rods.

32. A and B bought a horse for $40; A paid $25 and B $15; they afterward sold the horse for $64; how much should each receive? *Ans.* A $40 and B $24.

33. A has $12, B has $15, C $18, and D $24; what part of the whole has each one?

Ans. A, $\frac{4}{23}$; B, $\frac{5}{23}$; C, $\frac{6}{23}$, and D $\frac{8}{23}$.

34. What is the cost of 15 cwt. 2 qr. 15 lbs. sugar at 6 cts. per lb.? *Ans.* $93.90.

35. What per cent. is $\frac{1}{2}$? *Ans.* 50%.

36. What per cent. is $\frac{1}{4}$? *Ans.* 25%.

What per cent. is $\frac{1}{3}$? *Ans.* 33$\frac{1}{3}$%.

What per cent. is $\frac{2}{3}$? *Ans.* 66$\frac{2}{3}$%.

What per cent. is $\frac{1}{6}$? *Ans.* 16$\frac{2}{3}$%.

What per cent. is $\frac{1}{7}$? *Ans.* 14$\frac{2}{7}$%.

What per cent. is $\frac{2}{7}$? *Ans.* 28$\frac{4}{7}$%.

Rem. One-half of $\frac{100}{100}$ is $\frac{50}{100}$, or $\frac{1}{2}$ of 100% is 50%; one-fourth of $\frac{100}{100}$ is $\frac{25}{100}$, or $\frac{1}{4}$ of 100% is 25%; $\frac{100}{100}$, or 100%, is equal to 1 or a whole.

37. What is the interest of $240 for 5 years at 6%?

Ans. $72.

38. What is the interest of $634.50 for 3 years at 6%?

Ans. $114.21.

39. What is the interest of $760 for 4 years at 5%?

Ans. $152.

40. What is the interest of $836 for 3$\frac{1}{2}$ years at 4$\frac{1}{2}$%?

Ans. $131.67.

41. What is the interest of $556 for 8 months at 6% ? *Ans.* $22.24.

42. What is the interest of $864 for 9 months at 6% ? *Ans.* $38.88.

43. What is the interest of $8,425 for 16 months at 6% ? *Ans.* $674.

Rem. At 6% for any number of months, the rate % is one-half the number of months.

44. A and B traded together. A put in $1,200 for 16 months, and B $1,500 for 12 months; they gained $500; how much of the gain should each receive? *Ans.* A, 258\frac{2}{31}$; B, 241\frac{29}{31}$.

45. A farmer being asked how many sheep he had, replied that they were in four fields; in the first $\frac{1}{3}$ of the whole, in the second $\frac{1}{4}$, in the third $\frac{1}{6}$, and in the fourth 150 sheep; how many had he? *Ans.* 600 sheep.

46. If I buy coffee at 16 cts. per lb. and sell it for 20 cts., what % do I gain? *Ans.* 25%.

47. What is the present value of a note for $300, due in 8 months, bank discount, interest worth 6%, without grace? *Ans.* $288.

48. What are the proceeds of a note for $500, due in 3 months, discounted in bank, 3 days' grace? *Ans.* $492.25.

49. The proceeds of a note of $3,000 for 4 months, 6%, 3 days' grace, are? $2,938.50.

50. What is the bank discount of a note for $2,000, at 90 days, 6% ? *Ans.* $31.

51. A man spends $\frac{2}{3}$ of his income for board, $\frac{2}{3}$ of the remainder for clothing, and has $30 left; what is his income? *Ans.* $270.

52. If a man travel 39 mi. 7 fur. 8 rd. in 7 hours, what is the rate per hour? *Ans.* 5 mi. 5 fur. 24 rd.

53. The sum of two numbers is $\frac{6}{7}$, and their difference $\frac{1}{3}$; what are the numbers? *Ans.* $\frac{11}{42}$ and $\frac{25}{42}$.

54. If a quantity of provisions serve 1,200 men 12 months, how long will three times as many provisions serve 2,400 men? *Ans.* 18 months.

55. What will 12 A. 3 R. 20 P. of land cost at $30 per acre? *Ans.* $12\frac{7}{8} \times \$30 = \$386.25$.

56. What cost 35 A. 2 R. 10 P. at $20 per acre?
$35\frac{9}{16}$ A. *Ans.* $711.25.

57. A, B, and C together can do a piece of work in a certain time. A and B can do it in $\frac{7}{11}$ of the time, and B and C can do it in $\frac{8}{11}$ of the time. In what time can each one do it alone? *Ans.* A in $\frac{3}{11}$, B in $\frac{4}{11}$, C in $\frac{4}{11}$.

58. A can do a piece of work in $\frac{1}{3}$ of a day, B in $\frac{1}{4}$, and C in $\frac{1}{5}$; how long will it take all working together?
Ans. $\frac{1}{12}$.

59. How much grain must I take to mill, so as to have nine bushels remaining after the miller has taken one-tenth toll? *Ans.* 10 bu.

60. For what sum must I give my note, in bank, at 90 days, rate 6%, in order to receive three hundred dollars?
Int. $\frac{31}{2000}$, $\$300 = \frac{1969}{2000}$; $\therefore$ $\$300 \times \frac{2000}{1969} = \$304.72$.

61. By selling goods for $936.45 I lost 10%; had I gained 10%, what would I have sold them for?
Rate $\frac{110}{90}$. *Ans.* $1,144.55.

62. I have fifty gallons of wine, worth $1.50 per gallon; how much water must I add to reduce the price 25 cts. per gallon? *Ans.* 10 gals.

63. I bought 40 yds. of cloth at 5% less than cost, and

sold it at 10% above first cost, and gained $15; what was the first cost per yard?

Ans. $\frac{15}{100}$=$15, $\frac{100}{100}$=$100, $\frac{100}{40}$=$2.50.

Rem. Two terms form a ratio, and two equal ratios a proportion; thus,

64. As 3 is to 4, so is 6 to ? *Ans.* 8.
65. As 5 is to 7, so is 15 to ? *Ans.* 21.
66. As $\frac{1}{2}$ is to $\frac{2}{3}$, so is $\frac{3}{4}$ to ? *Ans.* 1.
67. As $\frac{2}{3}$ is to $\frac{3}{4}$, so is $\frac{16}{19}$ to ? *Ans.* $\frac{18}{19}$.
68. As $\frac{1}{2}$ is to $\frac{7}{8}$, so is $\frac{4}{9}$ to ? *Ans.* $\frac{7}{9}$.

Rem. 1. In any proportion the odd terms are antecedents, and the even terms consequents; each antecedent and consequent is termed a couplet; when one or more of the terms of a couplet are fractional, both terms must be reduced to a common denominator; and the numerators will express the ratio.

Rem. 2. A complete proportion has four terms, and if each consequent is made the numerator of a fraction and each antecedent a denominator, the fractions are equal, and may be put in the form of an equation; thus, if 3 : 4 :: 6 : 8, then $\frac{4}{3}=\frac{8}{6}$; they will also form an equation by alternating the terms; thus, $\frac{3}{4}=\frac{6}{8}$.
Let these equations, $\frac{4}{3}=\frac{8}{6}$ and $\frac{3}{4}=\frac{6}{8}$
be cleared of fractions by multiplying, $24=24$ $24=24$
each numerator by the other denominator; thus, and here we find that the product of the first and fourth, called the extremes, is equal to the product of the second and third, called the means, and by this principle, any three of the terms being known, the fourth is readily found.

80. If 13 men mow a field in 21 days, in what time would 39 men mow it? *Ans.* 7 days.

81. What is the difference between $21\frac{7}{9}$ and $20\frac{2}{3}$? *Ans.* $1\frac{1}{9}$.

82. Reduce $\frac{\frac{2}{3}}{\frac{3}{4}} \times \frac{\frac{4}{5}}{\frac{5}{6}}$ to a simple fraction. *Ans.* $\frac{64}{75}$.

83. Reduce to a simple fraction $\frac{\frac{4}{5}}{\frac{5}{6}} \div \frac{\frac{6}{7}}{\frac{5}{14}}$ *Ans.* $\frac{2}{5}$.

84. Reduce $\frac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}+\frac{4}{5}} \times \frac{\frac{5}{6}-\frac{2}{3}}{\frac{1}{2}-\frac{1}{4}}$ to a simple fraction. *Ans.* $\frac{140}{279}$.

85. Reduce $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \div \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8} \times \frac{8}{9}$ to a simple fraction. *Ans.* $\frac{9}{25}$.

86. Reduce $\frac{8 \times 12 \times 15 \times 18 \times 21 \times 24 \times 30}{20 \times 36 \times 42 \times 48}$ to a simple fraction. *Ans.* 270.

87. The product of two numbers is $\frac{5}{12}$, and one of the numbers is $\frac{2}{3}$; what is the other number? *Ans.* $\frac{5}{8}$.

88. The minuend is $\frac{8}{15}$, and the remainder is $\frac{2}{5}$; what is the subtrahend? *Ans.* $\frac{2}{15}$.

89. $\frac{2}{3}$ is how many times $\frac{2}{9}$? $\frac{2}{15}$? $\frac{2}{21}$? $\frac{2}{27}$? $\frac{2}{33}$? $\frac{2}{39}$? $\frac{2}{45}$? $\frac{2}{51}$?

90. $\frac{2}{5}$ is how many times $\frac{2}{15}$? $\frac{2}{25}$? $\frac{2}{35}$? $\frac{2}{45}$? $\frac{2}{55}$? $\frac{2}{65}$? $\frac{2}{75}$? $\frac{2}{85}$?

91. How many bottles of 1 qt. 1 pt. each can be filled from a hogshead (63 gals.) of wine? *Ans.* 168 bottles.

92. How many tiles 6 in. by 4 will cover a house 50 ft. long, and rafters on each side 24 ft. long? *Ans.* 14,400 shingles.

93. How many square yards of carpeting will cover a room 48 ft. long and 30 feet wide? *Ans.* 160 yds.

94. How many cubic yards of excavation in a mass of earth 90 ft. long, 30 ft. wide, and 24 ft. deep.
Ans. 2,400 cu. yd.

95. What is the cost of 1 T. 5 cwt. 65 lbs. beef at 8 cts. per lb.? 2,565 lb. *Ans.* $205.20.

96. How many cords of wood in a pile 28 ft. long, 8 ft. high, and 4 ft. wide? *Ans.* 7 cords.

97. How many acres in a rectangular tract of land 480 rods long and 60 rods wide?. *Ans.* 180 acres.

98. At 25 cents a gallon, how many gallons of beer can be bought for $57.25? *Ans.* 229 gallons.

99. Reduce 1,276 farthings to the higher denomination. £1 6s. 7d.

100. The discount of a note at 90 days, 6%, was $31; what was the face of the note given?

$\frac{31}{2000}$=$31, and $\frac{31\times2000}{31}$=the whole. *Ans.* $2,000.

101. A note with interest for 4 years 3 months, at 6%, was paid; the amount was $1,255; what was the face of the note?

$$\text{Int.}=\frac{25\frac{1}{2}}{100}=\frac{51}{200}=\text{principal and interest}=\frac{251}{200}=1255;$$

$\frac{1255\times200}{251}$. *Ans.* $1,000.

102. A farm of 375 A. 2 R. and 24 P. was sold at $51.50 per acre; what did the sale amount to?
375$\frac{13}{20}$ A. *Ans.* $19,345.97$\frac{1}{2}$.

103. How many rotations will a wheel 12 ft. in circumference make in a mile? *Ans.* 440 rotations.

104. How many barrels, each containing 2 bu. 3 pk., can be filled with 275 bu. of apples? *Ans.* 100 bar.

105. What is the difference between 25 lb. 8 oz. 9 dr. and 12 lb. 9 oz. 10 dr. of lead? *Ans.* 12 lb. 14 oz. 15 dr.

127. What is the interest of $4,000 for 636 days?
$\frac{2}{3} \times 636$. *Ans.* $424.

128. What is the interest of $1,000 for 1,248 days?
$\frac{1}{6} \times 1{,}248$. *Ans.* $208.

129. What is the interest of $2,000 for 624 days?
Ans. $208.

130. What is the interest of $60 for 3,000 days?
Ans. $30.

131. What is the interest of $742 for 3 y. 7 m. 24 d.?
Ans. $162.498.

132. What is the cost of 12 gal. 3 qt. 1 pt. wine at $3.00 per gal.? $12\frac{7}{8}$. *Ans.* $$38\frac{5}{8}$.

133. What cost 2 bu. 3 pk. 4 qt. apples at 80 cts. per bushel? $2\frac{7}{8}$ bu. *Ans.* $2.30.

134. The product is $8\frac{4}{5}$, and one of the factors is $3\frac{2}{5}$; what is the other factor? *Ans.* $2\frac{10}{17}$.

135. The dividend is $12\frac{3}{4}$, and the quotient $5\frac{2}{3}$; what is the divisor? *Ans.* $2\frac{1}{4}$.

136. The product of three factors is 4,125, and one factor is 15; what is the product of the other two?
Ans. $275.

137. Two men engage a piece of work; the one can do it in 5 days, and the other in $7\frac{1}{2}$ days; how long will it take both together? *Ans.* 3 days.

138. A tradesman hired a journeyman at 24 pence for every day he worked, and he was to forfeit 6d. for every idle day. At the end of 30 days his account balanced; how many days did he work? *Ans.* 6 days.

ANSWERS NOT GIVEN

IN THE PRECEDING PAGES.

ADDITION AND SUBTRACTION.

Page 16.

1. 115 sheep.
2. 145 fowls.
3. 63 in first and 21 in second.

Page 17.

4. 45 cattle in third field.
5. 28 cents left.
6. 139 pages to read.
7. 155 eggs.
8. 46 chickens.
9. 256 verses.
10. 190 cents.
11. 200 dollars.
12. 18830 dollars.
13. 13843 dollars left.

Page 18.

14. 14000 dollars.
15. 1193 acres left.
16. 888 dollars left.
17. 893 acres unsold, 247 acres of second farm, and 646 acres of third farm.

18. Sold for 467 dollars.
19. Gained 1118 dollars.
20. Gained $1182.
21. $3779.

MULTIPLICATION.

Page 26.

1. 18796796.
2. 12895164.
3. 32671178.
4. 17600100.
5. 505489116.
6. 3515971700.
7. 1913247450.
8. 8585904128.
9. 982259375.
10. 984381300.

DIVISION.

Page 31.

1 Quo. 1915, rem. 98.
2. " 1477, " 60.
3. " 263, " 234.
4. " 5475, " 116.
5. " 209205, " 992.
6. " 780317, " 300.
7. Quo. 800368, rem. 209.
8. " 373792, " 979.
9. " 439169, " 1458.
10. " 13482, " 42.
11. " 4457, " 102170.
12. " 676, " 196432.

1. 50 cts.
2. 5 lbs.
3. 100 cents.
4. 10 lbs.
5. 150 cents.
6. 15 lbs.
7. 200 cts.
8. 20 lbs.
9. 250 cts.
10. 25 lbs.

Page 32.

11. 1960 cents.
12. 245 lbs.
13. $15660.
14. 348 acres.
15. $207680.
16. 3245 acres.

17. $1875.
18. 15 horses.
19. $2625.
20. 35 oxen.
21. $3780.
22. 84 cows.

Page 33.

32. Horses cost $2016, cows $1944, sheep $1456, all cost $5416.

33. $5740.

34. Cost $76600, sold for $75355.

35. Gained $6912.

Page 34.

36. Paid $25 per acre, and sold for $29 per acre.

37. Paid $65 per acre, sold for $71.

38. Each cow cost $40.

39. *1st Ans.*, 36000 farms; *2d Ans.*, 184000 farms; *3rd Ans.*, 1028000 farms.

40. 1st State, $144,000,000; 2d State, $588,800,000; 3rd State, $822,400,000.

41. Each daughter's share $7100.

FACTORING.

Page 50.

1. 60 = 2, 2, 3, 5.
2. 64 = 2, 2, 2, 2, 2, 2.
3. 65 = 5, 13.
4. 70 = 2, 5, 7.
5. 72 = 2, 2, 2, 3, 3.
6. 75 = 3, 5, 5.
7. 78 = 2, 3, 13.
8. 80 = 2, 2, 2, 2, 5.
9. 84 = 2, 2, 3, 7.
10. 86 = 2, 43.
11. 88 = 2, 2, 2, 11.
12. 90 = 2, 3, 3, 5.
13. 95 = 5, 19.
14. 96 = 2, 2, 2, 2, 2, 3.
15. 98 = 2, 7, 7.
16. 100 = 2, 2, 5, 5.

17. 102 = 2, 3, 17.
18. 104 = 2, 2, 2, 13.
19. 106 = 2, 53.
20. 108 = 2, 2, 3, 3, 3.
21. 110 = 2, 5, 11.
22. 112 = 2, 2, 2, 2, 7.
23. 114 = 2, 3, 19.
24. 116 = 2, 2, 29.
25. 118 = 2, 59.
26. 120 = 2, 2, 2, 3, 5.

27. 124 = 2, 2, 31.
28. 125 = 5, 5, 5.
29. 128 = 2, 2, 2, 2, 2, 2, 2.
30. 130 = 2, 5, 13.
31. 132 = 2, 2, 3, 11.
32. 136 = 2, 2, 2, 17.
33. 140 = 2, 2, 5, 7.
34. 225 = 5, 5, 9.
35. 500 = 2, 2, 5, 5, 5.
36. 625 = 5, 5, 5, 5.

MULTIPLICATION OF FRACTIONS.

Page 63.

7. $\frac{44}{3} \times 4 = \frac{176}{3} = 58\frac{2}{3}$.
8. $\frac{86}{5} \times 6 = \frac{516}{5} = 103\frac{1}{5}$.
9. $\frac{73}{4} \times 9 = \frac{657}{4} = 164\frac{1}{4}$.
10. $\frac{99}{\cancel{4}} \times \overset{2}{\cancel{8}} = 198$.
11. 22204.
12. $64916\frac{2}{3}$.
13. $3147 \times \frac{143}{4} = \frac{450021}{4} = 112505\frac{1}{4}$.
14. $\frac{16627}{\cancel{4}} \times \frac{\overset{\overset{532}{\cancel{15961}}}{\cancel{6384}}}{\cancel{31}} = 16627 \times 532 = 8845564$.
15. $\frac{1402}{11} \times \frac{94}{7} = \frac{131788}{77} = 1711\frac{41}{77}$.
16. $\frac{139}{9} \times \frac{65}{7} = \frac{9035}{63} = 143\frac{26}{63}$.
17. $\frac{1076}{37} \times \frac{275}{21} = \frac{295900}{777} = 380\frac{640}{777}$.
18. $\frac{314}{3} \times \frac{184}{9} = \frac{57776}{27} = 2139\frac{23}{27}$.

DIVISION OF FRACTIONS.

Page 65.

1. $\overset{9}{\cancel{36}} \times \frac{5}{\cancel{4}} = 45.$

2. $\overset{6}{\cancel{12}} \times \frac{3}{\cancel{2}} = 18.$

3. $\overset{4}{\cancel{16}} \times \frac{7}{\cancel{4}} = 28.$

4. $\frac{3}{\underset{2}{\cancel{4}}} \times \frac{\overset{3}{\cancel{6}}}{5} = \frac{9}{10}.$

5. $\frac{1}{2} \times \frac{3}{2} = \frac{3}{4}.$

6. $\frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}.$

7. $\frac{\overset{45}{\cancel{225}}}{2} \times \frac{3}{\underset{13}{\cancel{65}}} = \frac{135}{26} = 5\frac{5}{26}.$

8. $\frac{\overset{4}{\cancel{12}}}{\cancel{3}} \times \frac{\overset{2}{\cancel{6}}}{\underset{5}{\cancel{15}}} = \frac{8}{5} = 1\frac{3}{5}.$

9. $\frac{100}{\cancel{3}} \times \frac{\cancel{3}}{19} = \frac{100}{19} = 5\frac{5}{19}.$

10. $15 \times \frac{15}{7} = \frac{225}{7} = 32\frac{1}{7}.$

11. $\frac{15}{4} \times \frac{15}{7} = \frac{225}{28} = 8\frac{1}{28}.$

12. $\frac{15}{19} \times \frac{7}{2} = \frac{105}{38} = 2\frac{29}{38}.$

13. $\frac{13}{15} \times \frac{11}{9} = \frac{143}{135} = 1\frac{8}{135}.$

14. $\frac{835}{4} \times \frac{9}{248} = \frac{7515}{992} = 7\frac{771}{992}.$

15. $\frac{873}{8} \times \frac{7}{205} = \frac{6111}{1640} = 3\frac{1191}{1640}.$

Page 75.

1. $\frac{2}{3}$ of $\frac{3}{16} = \frac{1}{8}$; ∴ $\frac{1}{8} =$ $50,000, and the whole is eight times $\frac{1}{8} =$ $400,000.

2. They will meet in $7\frac{3}{55}$ hours, A will have traveled $45\frac{47}{55}$ miles, and B $51\frac{8}{55}$ miles.

DENOMINATE NUMBERS.

Page 90.

4. £291 15*s.* 9*d.*

Page 91.

2. *Ans.*, 500 cents; 5000 mills.
3. 700 cents; 7000 mills.
4. 515 cents; 5150 mills. 5. 6153 mills.

Page 92.

9. 63 dollars 25 cents 7 mills; $63.257.
10. $753.25. 11. $9.00. 12. $10.50.
13. $105. 14. $1.125. 15. $39.375.
16. $55.65.
17. Cost $483.136; sold for $640.328.

Page 93.

24. 9303 far. 26. £39 19s. 3d. 3 far.
25. £39442 1 far. 27. £1 19s. 2d. $\frac{2}{13}$ far.

Page 102.

1. 3359 far. 7. 1705577 dr.
2. 3068d. 8. 1 T. 6 cwt. 1 qr 1 lb. 11 oz.
3. 1682d. 9. 32871 qr.
4. £2 9s. 2 far. 10. 11 lb. 2 oz. 4 qr.
5. £2 5s. 3d. 11. 14690 gr.
6. £36 11s. 12. 1 lb. 2 ℥ 1 ℈ 2 gr.
13. 741402 in.
14. 4 Leagues 5 fur. 12 rods 4 yd. 1 ft. 4 in.
15. 177½ inches. 18. 162 inches.
16. 135 inches. 19. 172048 sq. in.
17. 180 inches. 20. 161940 cu. in.
21. 2 cords.

Page 103.

22. 831 gills. 23. 633 pts.

Page 106.

56. 1440 yds.; 1200 yds.; 960 yds.; 640 yds.; 480 yds.; 384 yds.; 320 yds.; $274\frac{2}{7}$ yds.

57. Proceeds \$3312.50 and $3785\frac{4}{7}$ yds.

RATIO.

Page 113.

5. 16 days. 6. 120 feet.

Page 114.

7. \$25.

9. \$10.50; \$11.40; \$28.50; \$38; \$57; \$47; \$70.50; \$117; \$128.10.

10. $\$11.87\frac{1}{2}$; $\$12.81\frac{1}{4}$; \$19.50; \$46.50; \$119.50; \$157; \$188.50; \$349.25.

11. \$260; \$310; \$500; \$525; \$690; \$735; \$780; \$825; \$1510; \$1995.

12. \$37.50; \$25; \$12.50; \$10; \$20; \$30; \$40; \$60; \$70.

13. $\$\frac{108}{175}$. 14. \$9. 15. \$18.75 and \$22.50.

Page 116.

3. *Ans.*, 9 days.

PERCENTAGE.

Page 131.

5. $500. 6. 13335 Francs. 7. $10,000.

EVOLUTION.

Page 147.

10. 2700 cu. ft. 12. 9.4248 cu. ft.
14. 21.2058 cu. ft.

[redacted]

[illegible] OF 25 CENTS
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